Solar flares, current sheets, and finite-time singularities in Hall magnetohydrodynamics

Yuri Litvinenko

Department of Mathematics, University of Waikato, New Zealand

Outline

- Reconnecting current sheets in the flaring solar corona
- Self-similar solutions for current sheet formation in 2D magnetohydrodynamics (MHD) and Hall MHD
- Finite-time singularities in Hall MHD

Energy release in flares

Flare:
impulsive release of magnetic energy as radiation, heat, fast flows and particles in the lower corona of the Sun.

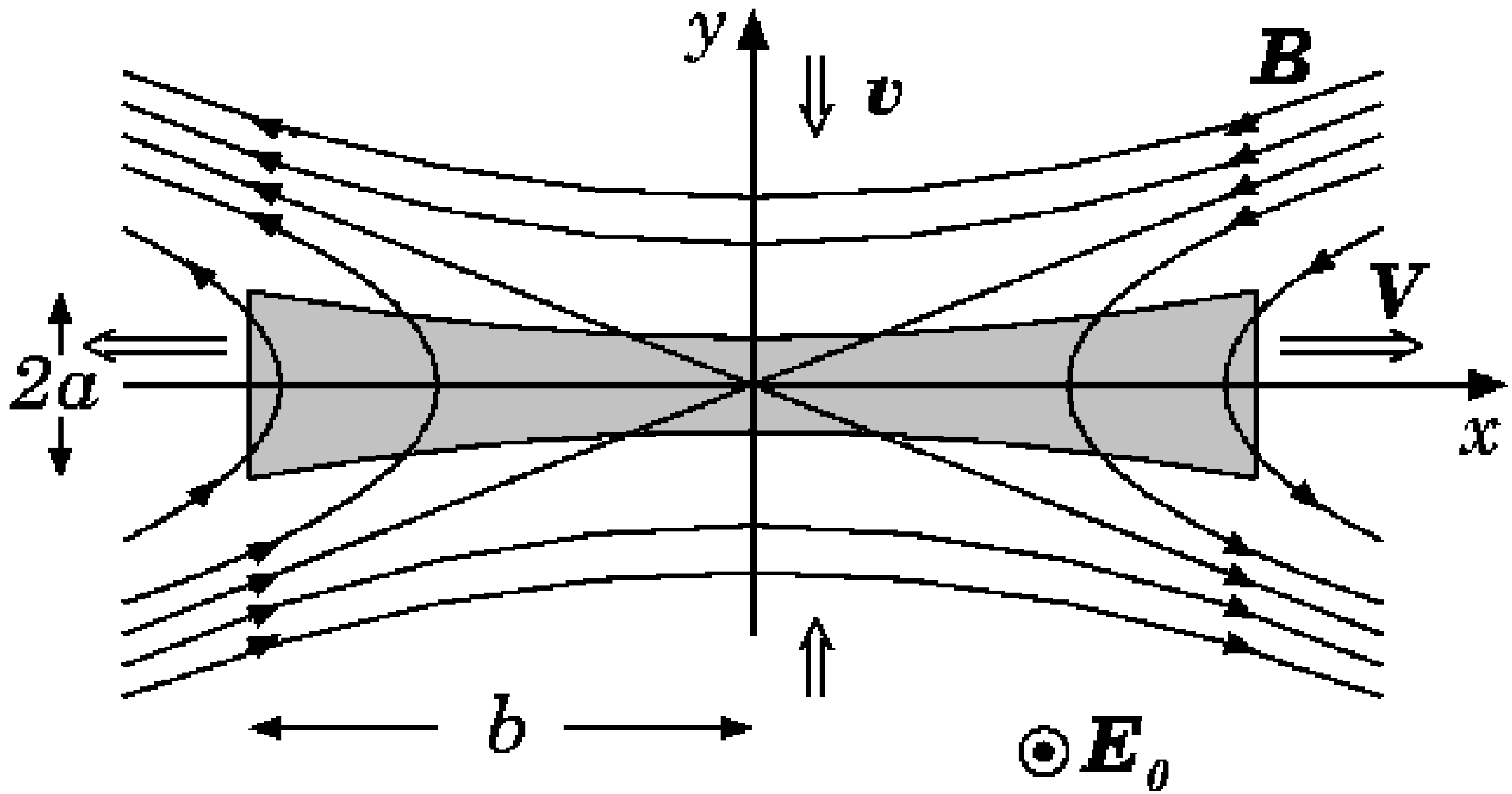
Total flare energy up to $\mathcal{E} \simeq 10^{32}$ erg:

$$B = 100 \text{ G}, \quad L = 10^{10} \text{ cm} \implies \frac{B^2}{8\pi} L^3 \geq \mathcal{E}$$

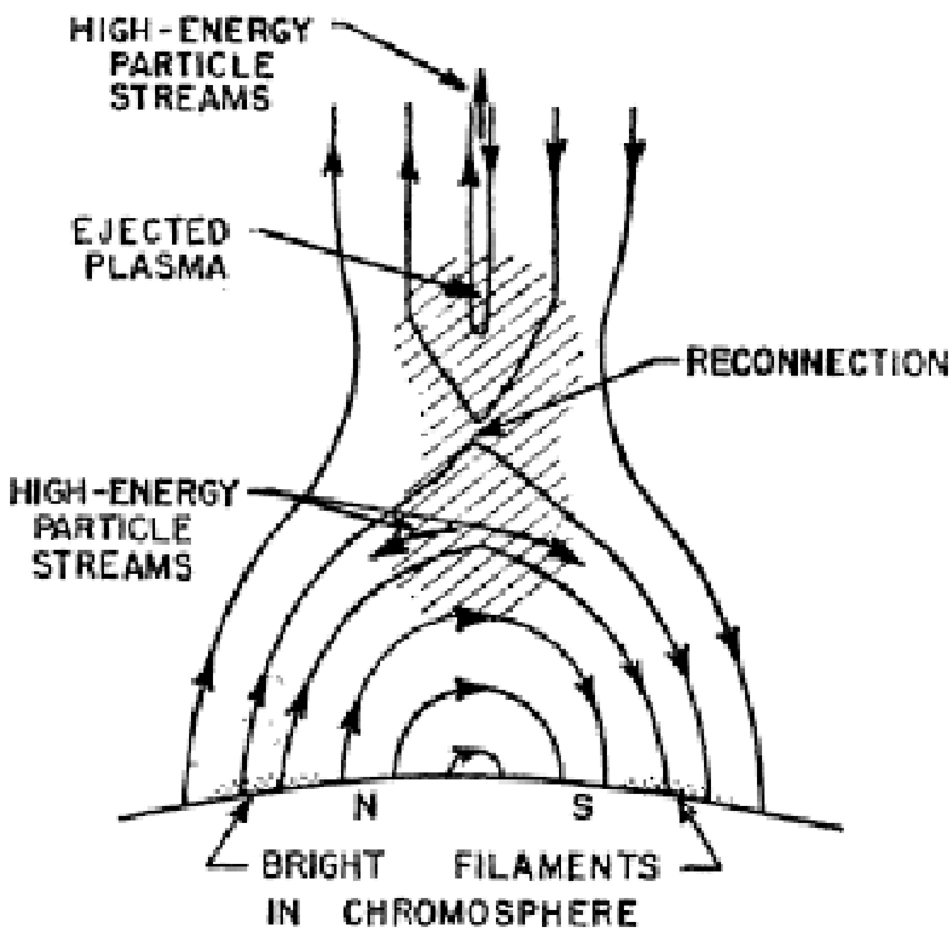
Flare time $t_f \simeq 10^3$ s:

$$t_f \leq 100 \; t_A, \quad t_A = \frac{L}{v_A}, \quad v_A = \frac{B}{\sqrt{4\pi\rho}}$$

Reconnecting current sheet

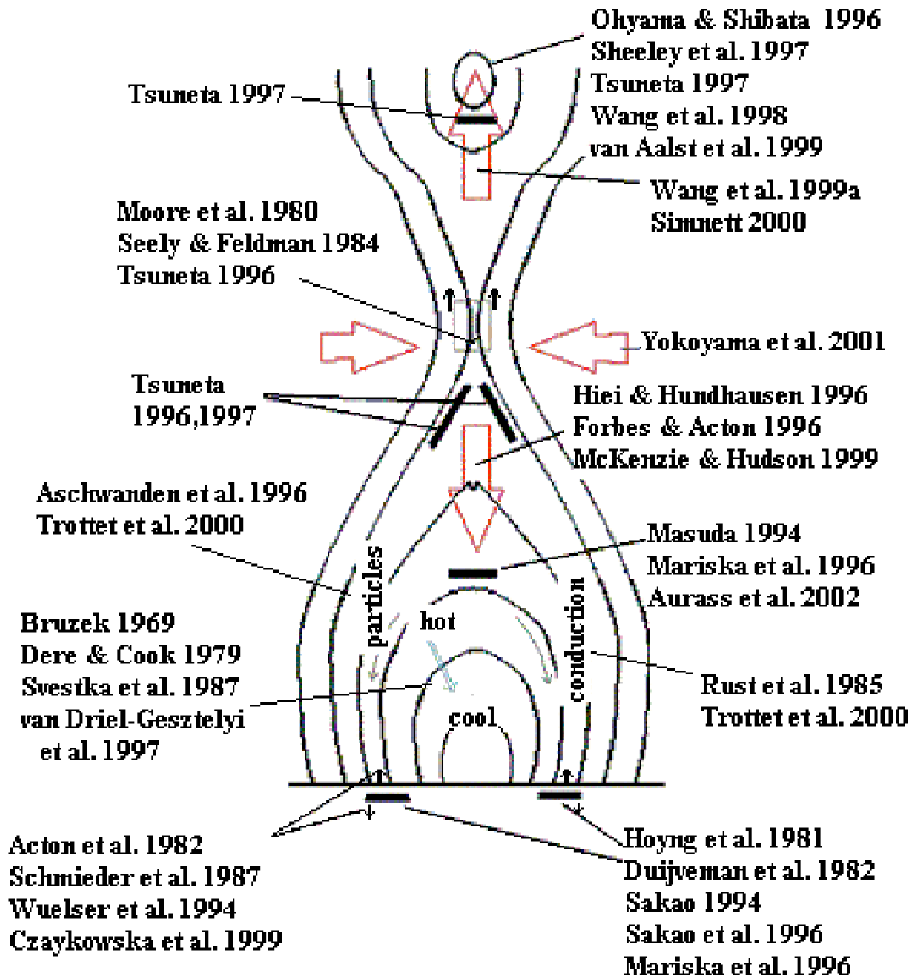


Solar flare geometry



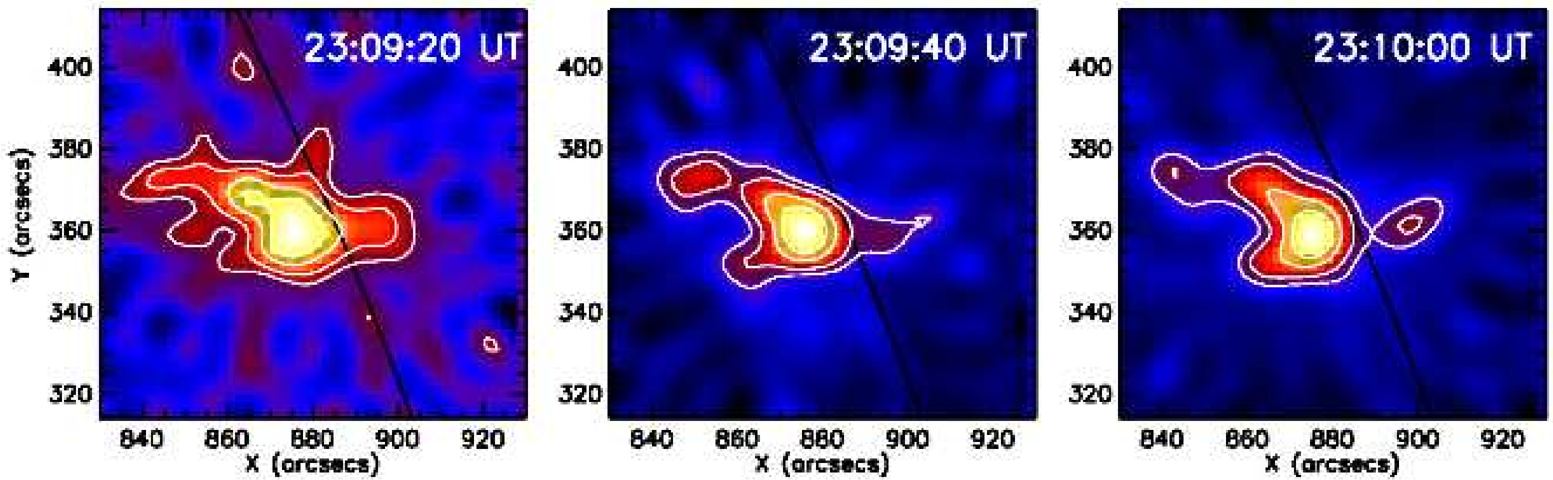
(Sturrock, 1968)

Observational evidence for reconnection



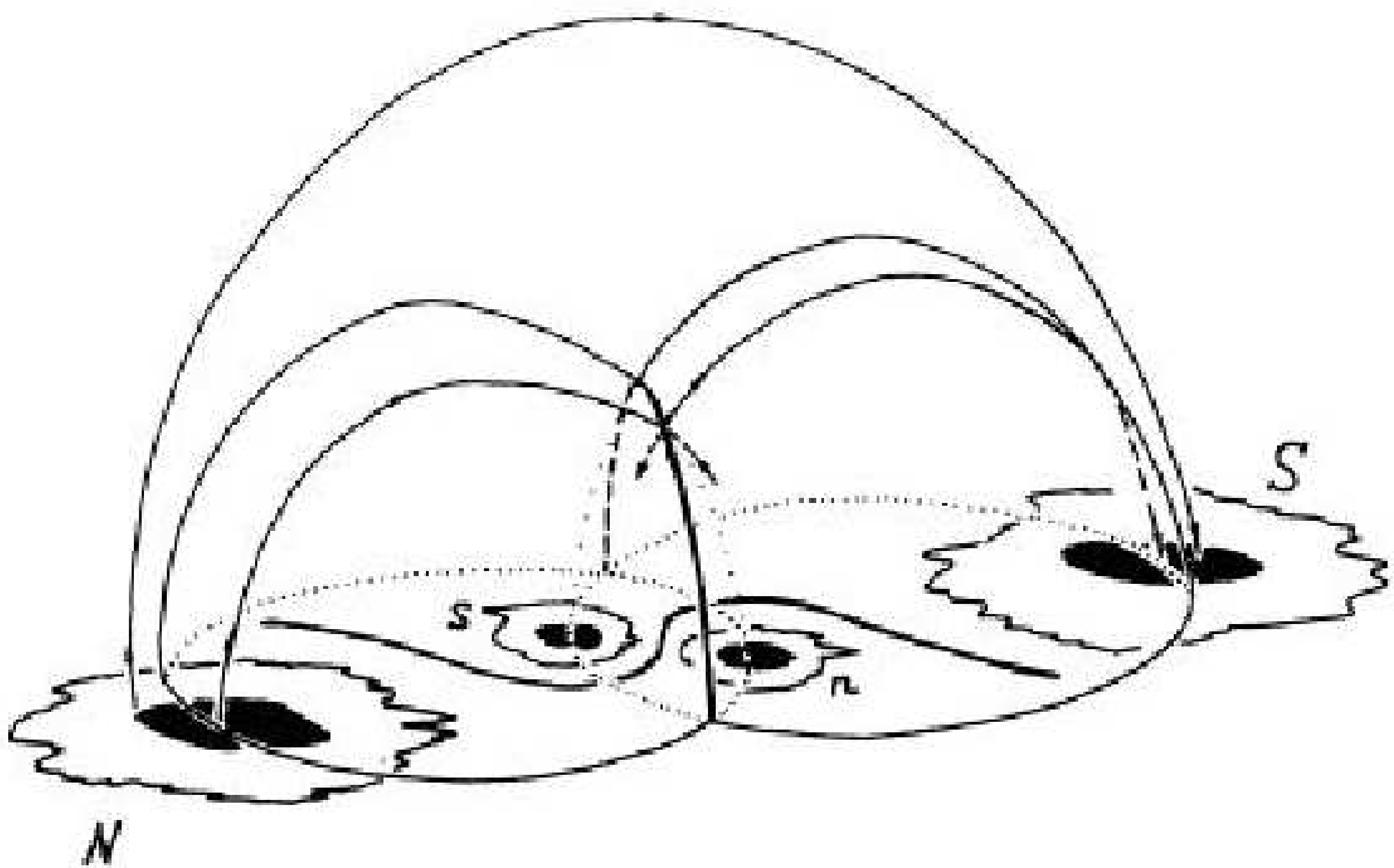
(McKenzie, 2001)

RHESSI X-ray images of a current sheet



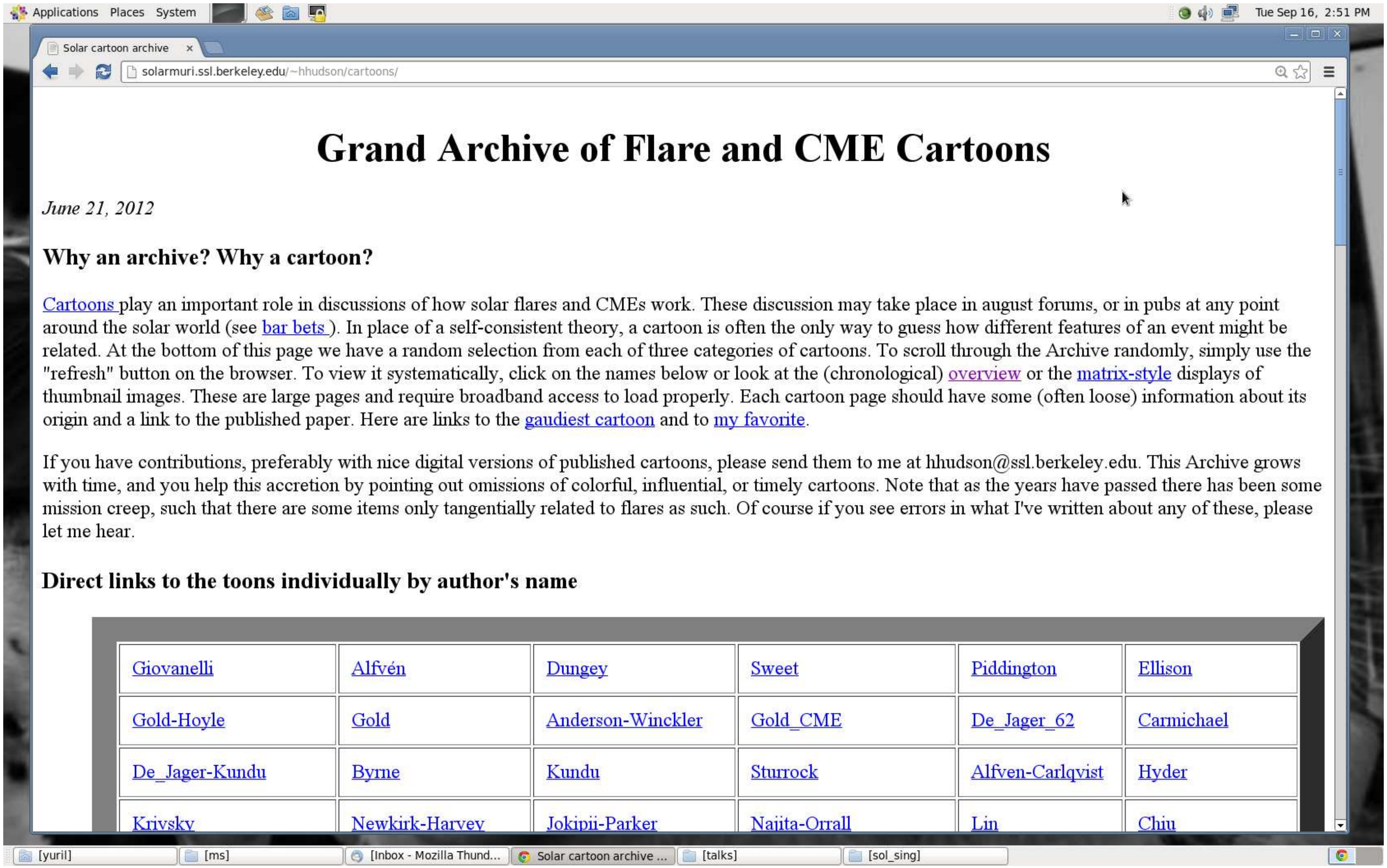
(Sui and Holman, 2003)

Flare geometry in three dimensions



(Machado et al., 1983)

Hugh Hudson’s archive of flare cartoons



Grand Archive of Flare and CME Cartoons

June 21, 2012

Why an archive? Why a cartoon?

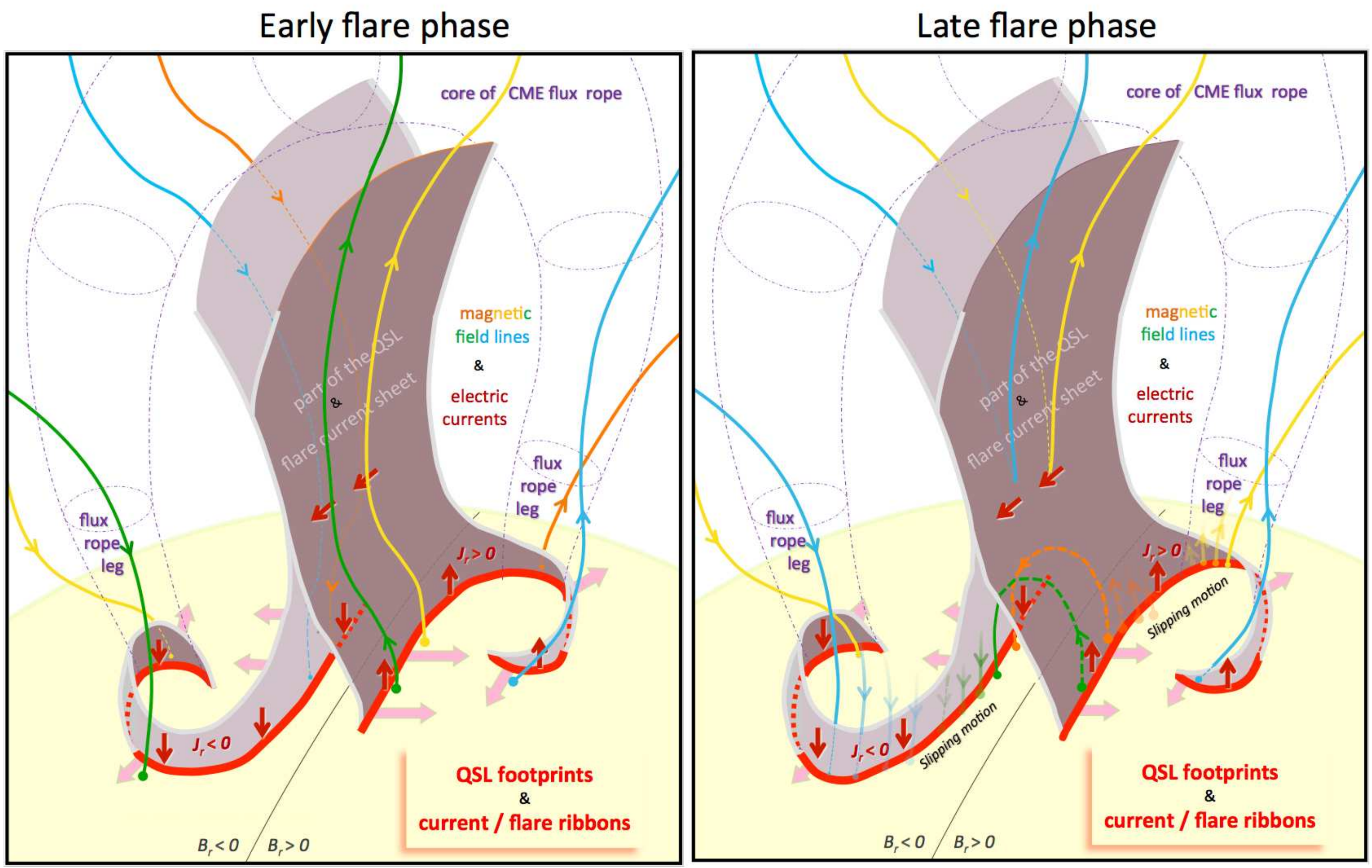
Cartoons play an important role in discussions of how solar flares and CMEs work. These discussion may take place in august forums, or in pubs at any point around the solar world (see bar bets). In place of a self-consistent theory, a cartoon is often the only way to guess how different features of an event might be related. At the bottom of this page we have a random selection from each of three categories of cartoons. To scroll through the Archive randomly, simply use the "refresh" button on the browser. To view it systematically, click on the names below or look at the (chronological) overview or the matrix-style displays of thumbnail images. These are large pages and require broadband access to load properly. Each cartoon page should have some (often loose) information about its origin and a link to the published paper. Here are links to the gaudiest cartoon and to my favorite.

If you have contributions, preferably with nice digital versions of published cartoons, please send them to me at hhudson@ssl.berkeley.edu. This Archive grows with time, and you help this accretion by pointing out omissions of colorful, influential, or timely cartoons. Note that as the years have passed there has been some mission creep, such that there are some items only tangentially related to flares as such. Of course if you see errors in what I've written about any of these, please let me hear.

Direct links to the toons individually by author's name

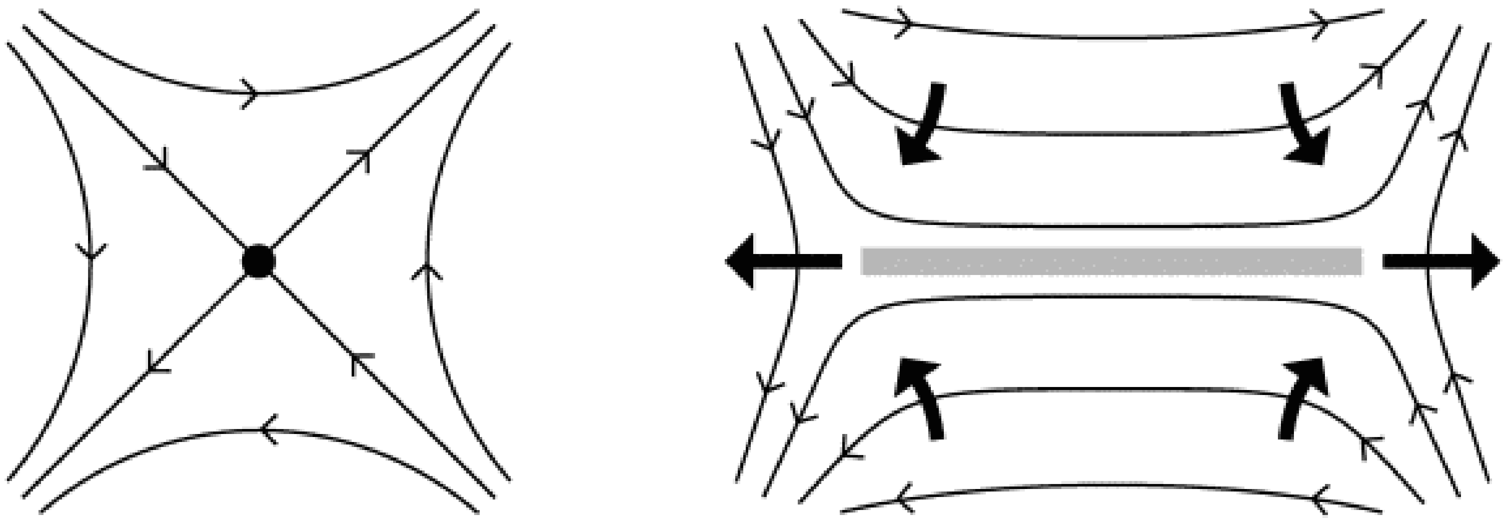
Giovanelli	Alfvén	Dungey	Sweet	Piddington	Ellison
Gold-Hoyle	Gold	Anderson-Winckler	Gold_CME	De_Jager_62	Carmichael
De_Jager-Kundu	Byrne	Kundu	Sturrock	Alfven-Carlqvist	Hyder
Krivsky	Newkirk-Harvey	Jokipii-Parker	Najita-Orrall	Lin	Chiu

A recent flare cartoon



(Janvier, 2014)

Current sheet formation at a neutral line



Parameters of a solar active region

Typical values:

$$L = 10^{9.5} \text{ cm}, \quad T = 10^{6} \text{ K}, \quad \rho = 10^{-15} \text{ g cm}^{-3},$$

$$B = 10^{2} \text{ G}, \quad v_A = \frac{B}{\sqrt{4\pi\rho}} \simeq 10^{9} \text{ cm s}^{-1}$$

Dimensionless resistivity:

$$\eta = \frac{c^2}{4\pi v_A L \sigma} \simeq 10^{-14.5}$$

Dimensionless ion inertial length:

$$d_i = \frac{c}{L\omega_{pi}} \simeq 10^{-6.5} > \eta^{1/2}$$

Dimensionless equations of Hall MHD

Momentum equation:

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mathbf{J} \times \mathbf{B}$$

Ohm's law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = d_i(\mathbf{J} \times \mathbf{B} - \nabla p_e)$$

Incompressibility:

$$\nabla \cdot \mathbf{v} = 0$$

Maxwell's equations:

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad \mathbf{J} = \nabla \times \mathbf{B}$$

$2\frac{1}{2}$D Hall MHD solution

$$\mathbf{v}(x,y,t) = \nabla\phi \times \hat{\mathbf{z}} + W\hat{\mathbf{z}}$$

$$\mathbf{B}(x,y,t) = \nabla\psi \times \hat{\mathbf{z}} + Z\hat{\mathbf{z}}$$

Planar components:

$$\partial_t(\nabla^2\phi) + [\nabla^2\phi, \phi] = [\nabla^2\psi, \psi]$$

$$\partial_t\psi + [\psi, \phi] = d_i[\psi, Z]$$

Axial components:

$$\partial_t W + [W, \phi] = [Z, \psi]$$

$$\partial_t Z + [Z, \phi] = [W, \psi] + d_i[\nabla^2\psi, \psi]$$

A self-similar solution

Stream function:

$$\phi = -\gamma(t)xy$$

Flux function:

$$\psi = \alpha(t)x^2 - \beta(t)y^2$$

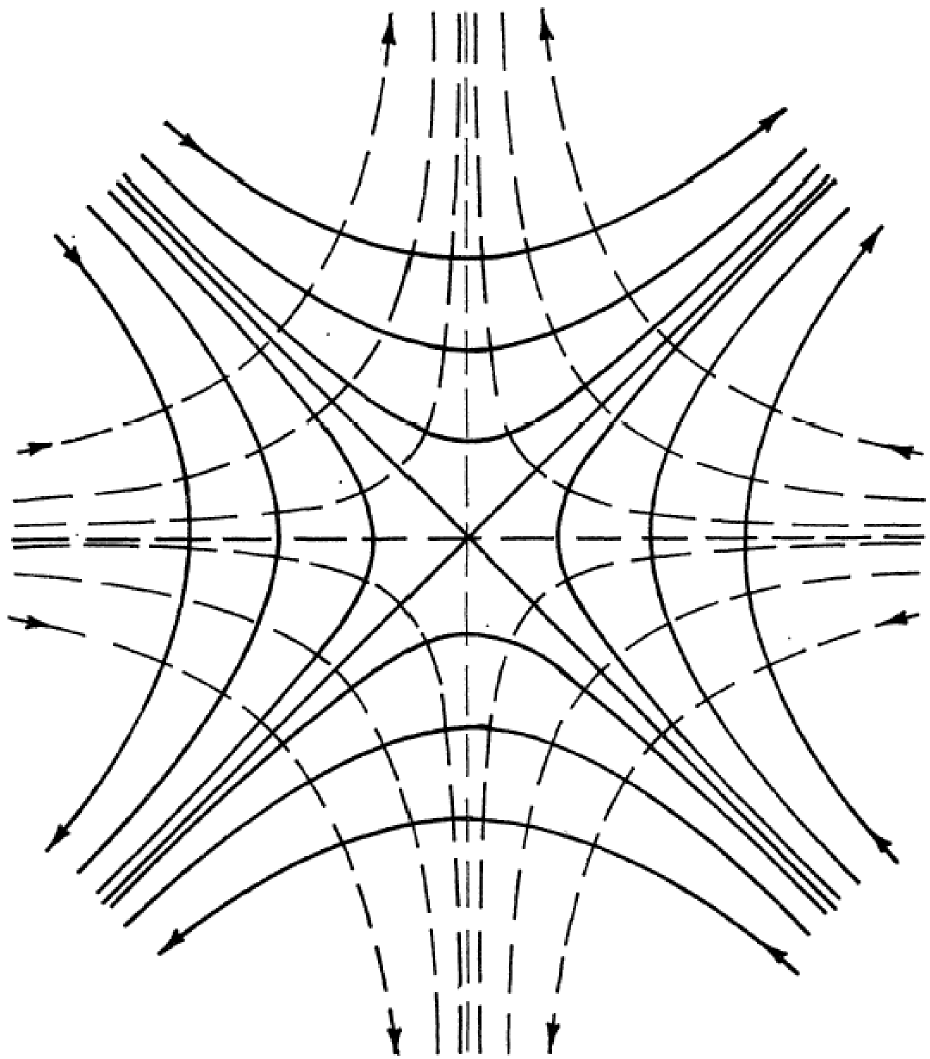
Axial speed:

$$W = f(t)x^2 + g(t)y^2$$

Axial magnetic field:

$$Z = h(t)xy$$

v and **B** at $t = 0$



Reconnection in a resistive viscous plasma

$\eta \neq 0,\ \nu \neq 0 \Longrightarrow$

$$\psi \to \psi + 2\eta \int (\alpha - \beta) dt$$

$$W \to W + 2\nu \int (f + g) dt$$

The similarity reduction (a system of ODEs)

$$\dot{\alpha} - 2\alpha\gamma - 2d_i\alpha h = 0$$

$$\dot{\beta} + 2\beta\gamma + 2d_i\beta h = 0$$

$$\dot{f} - 2\gamma f + 2\alpha h = 0$$

$$\dot{g} + 2\gamma g + 2\beta h = 0$$

$$\dot{h} + 4\alpha g + 4\beta f = 0$$

Current sheet formation in 2D MHD

$$d_i = 0$$

$$f = g = h = 0, \quad \gamma = \gamma_0$$

$$\dot{\alpha} - 2\alpha\gamma_0 = 0$$

$$\dot{\beta} + 2\beta\gamma_0 = 0$$

Exponential growth, no finite-time singularities (FTS):

$$\alpha(t) = \exp(2\gamma_0 t)$$

$$\beta(t) = \exp(-2\gamma_0 t)$$

(Chapman and Kendall, 1963; Sulem et al., 1985; Grauer and Marliani, 1998)

Effect of the Hall term on the magnetic field

$$d_i > 0$$

$t \ll 1$:

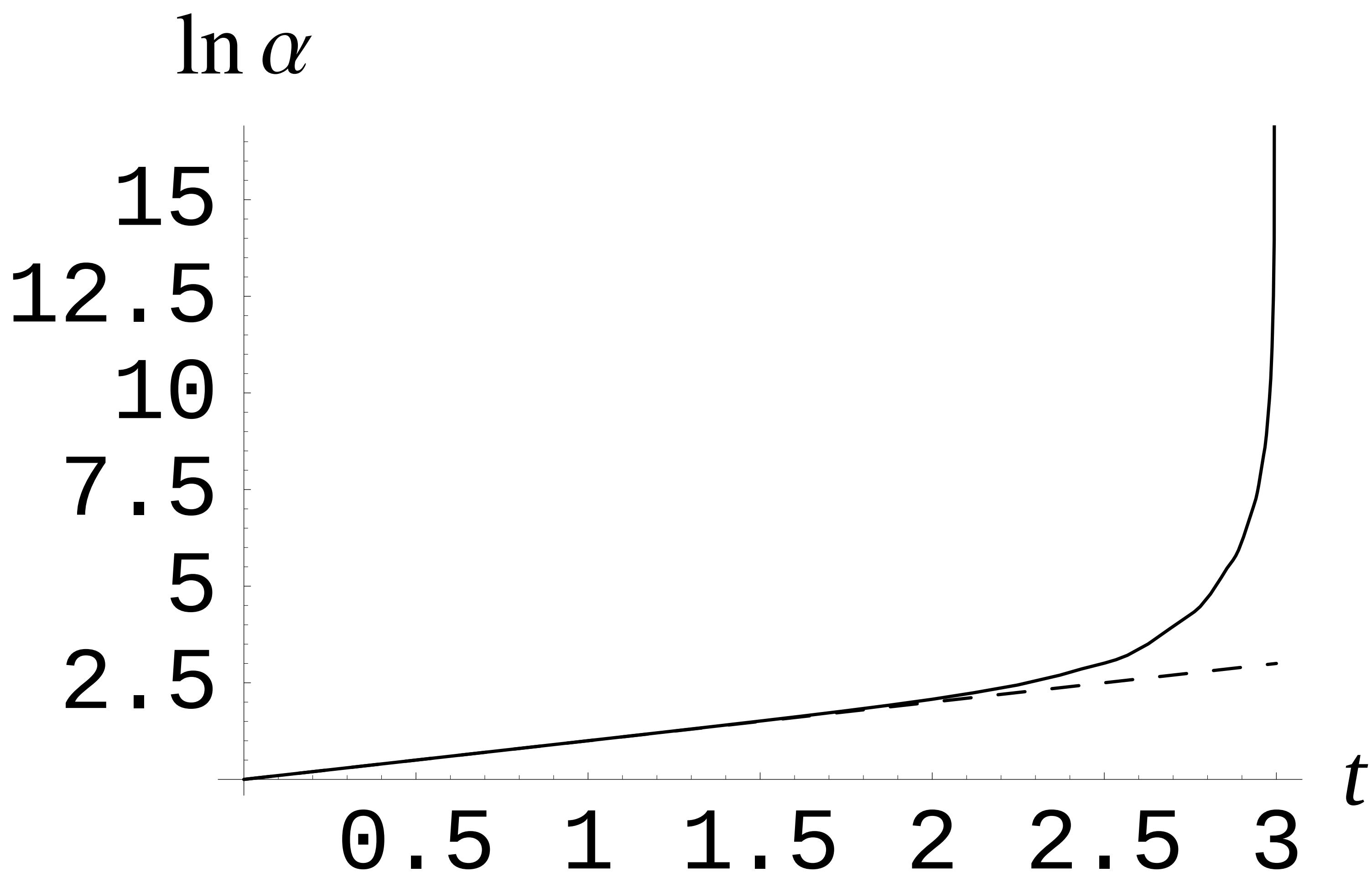
$$\alpha(t) \approx 1 + (2\gamma_0 + 2d_i h_0)t$$

$$\beta(t) \approx 1 - (2\gamma_0 + 2d_i h_0)t$$

The angle between the magnetic separatrices, $\psi = 0$:

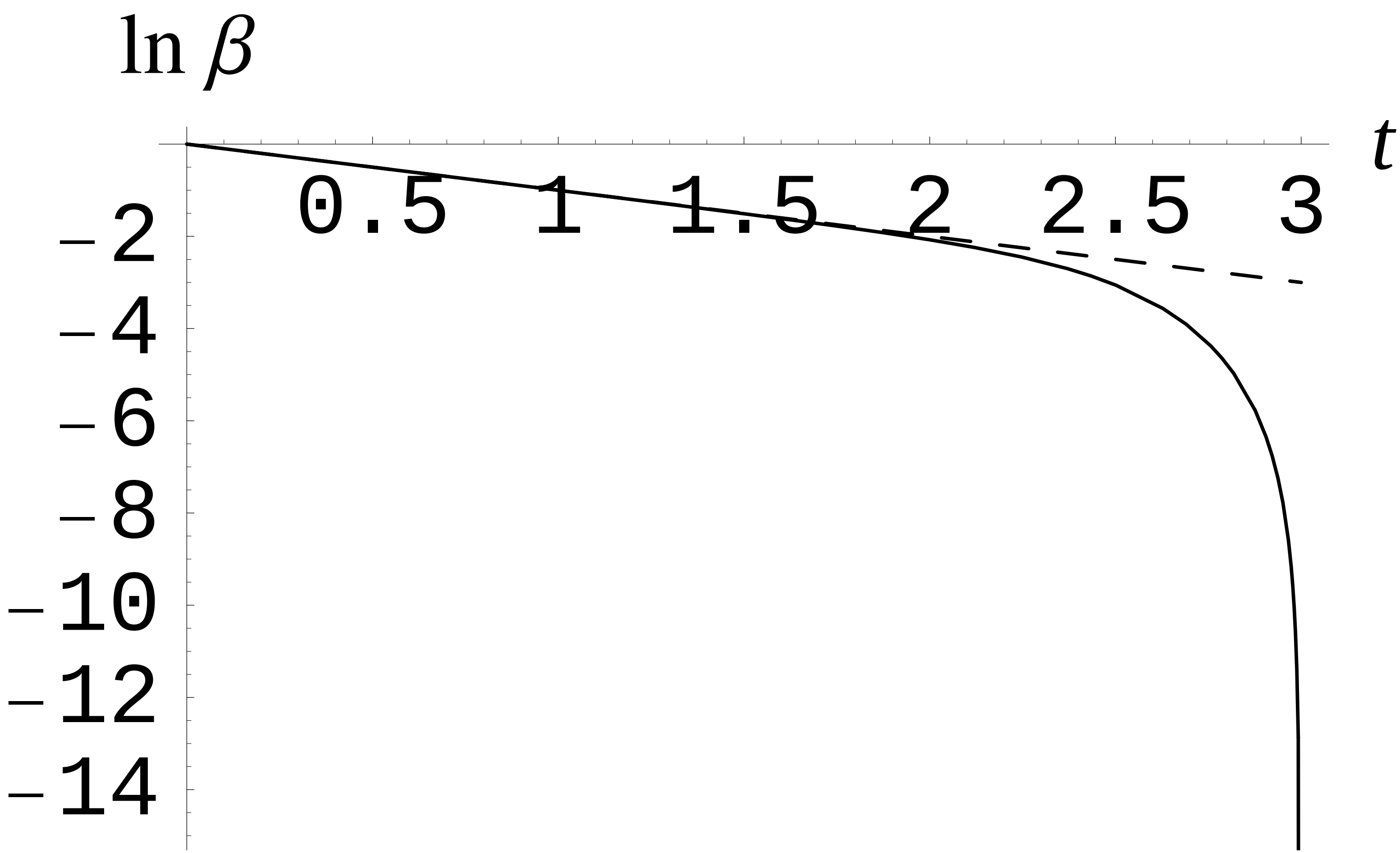
$$\tan\frac{\theta}{2} = \left(\frac{\beta}{\alpha}\right)^{1/2} \approx 1 - (2\gamma_0 + 2d_i h_0)t$$

Collapse to a current sheet: $d_i h_0 = 10^{-4}$



$\alpha(0) = \beta(0) = 1, \gamma(0) = \gamma_0, f(0) = g(0) = 0$

Collapse to a current sheet: $d_i h_0 = 10^{-4}$



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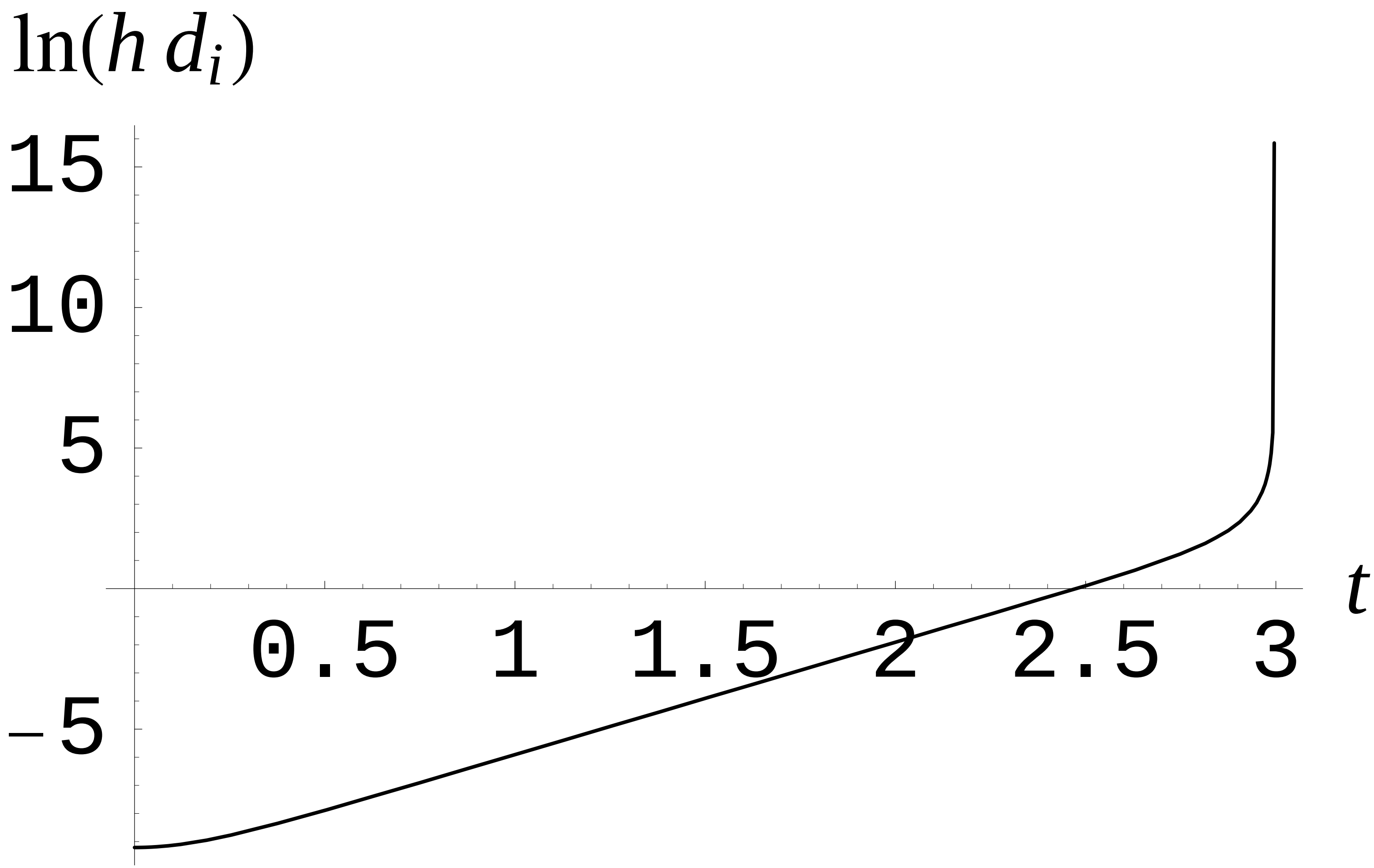
Collapse to a current sheet: $d_i h_0 = 10^{-4}$



$\alpha(0) = \beta(0) = 1,\ \gamma(0) = \gamma_0,\ f(0) = g(0) = 0$

Finite-time singularity: a mechanical analogy

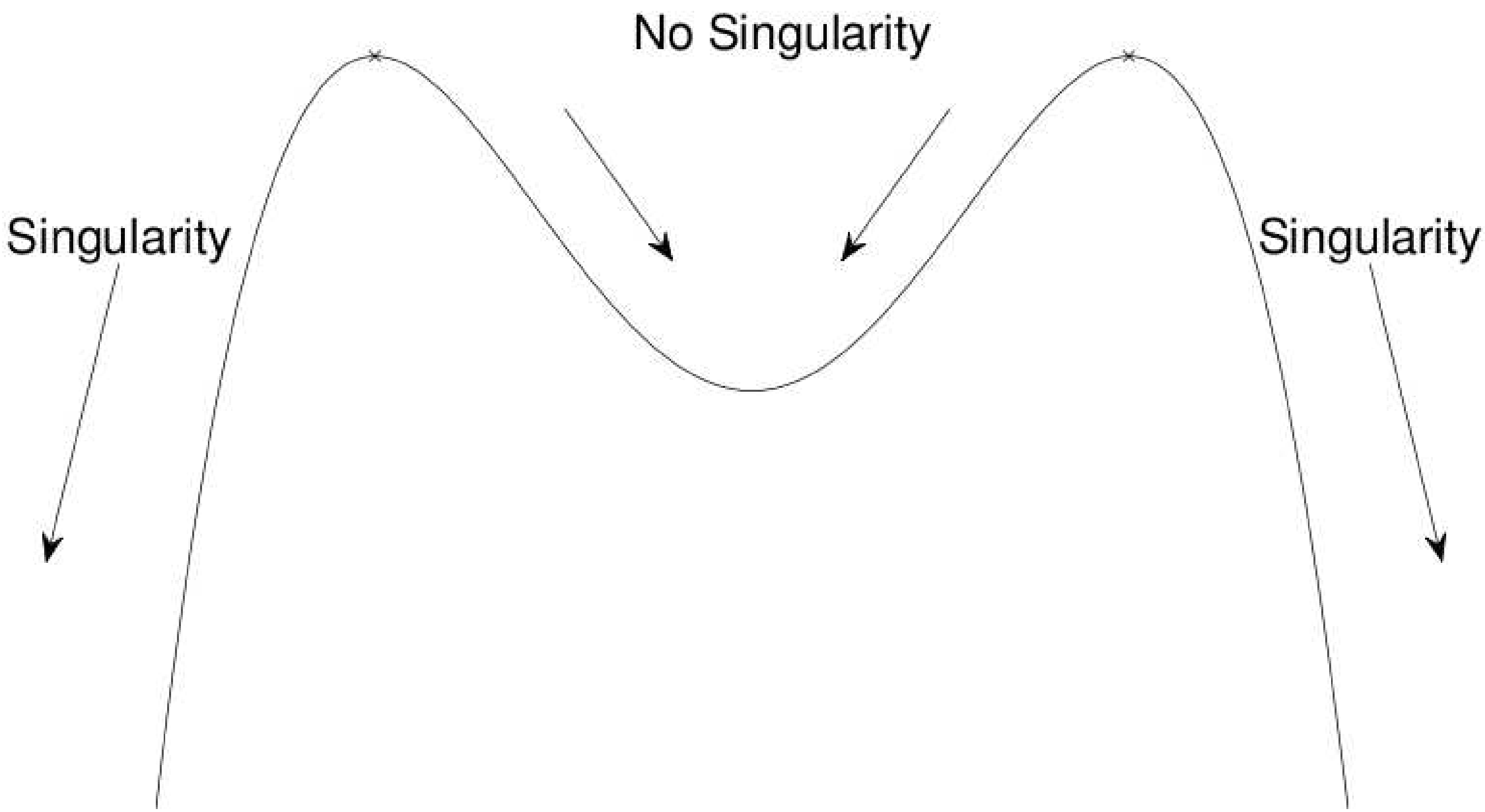
FTS: $h(t) \to \infty$ as $t \to t_s$

$$\ddot{h} + U'(h) = 0$$

$$U(h) = -\frac{1}{2}(d_i^2 h^4 + a^2 h^2) + \frac{1}{2}(d_i^2 h_0^4 + a^2 h_0^2) - 8(\alpha_0 g_0 + \beta_0 f_0)^2$$

$$a^2 = -2[4d_i(\alpha_0 g_0 - \beta_0 f_0) - 8\alpha_0\beta_0 + d_i^2 h_0^2]$$

Finite-time singularity: a mechanical analogy



Criterion for the FTS absence

$$\alpha_0\beta_0(\alpha_0 + d_i f_0)(\beta_0 - d_i g_0) \geq 0$$

$$d_i(\alpha_0 g_0 - \beta_0 f_0) - 2\alpha_0\beta_0 \geq 0$$

(Litvinenko and McMahon, 2015)

Behaviour near the singularity ($h > 0$)

$$\tau = (t_s - t) \to 0, \quad \Gamma = \int_0^t \gamma(t')dt' \to \Gamma_s$$

$$\alpha \approx \frac{1}{4(\beta_0 - d_i g_0)} \exp(2\Gamma_s)\tau^{-2}$$

$$\beta \approx 4\alpha_0\beta_0(\beta_0 - d_i g_0) \exp(-2\Gamma_s)\tau^2$$

$$d_i f \approx -\frac{1}{4(\beta_0 - d_i g_0)} \exp(2\Gamma_s)\tau^{-2}$$

$$d_i g \approx -(\beta_0 - d_i g_0) \exp(-2\Gamma_s)$$

$$d_i h \approx \tau^{-1}$$

An intermediate asymptotic solution

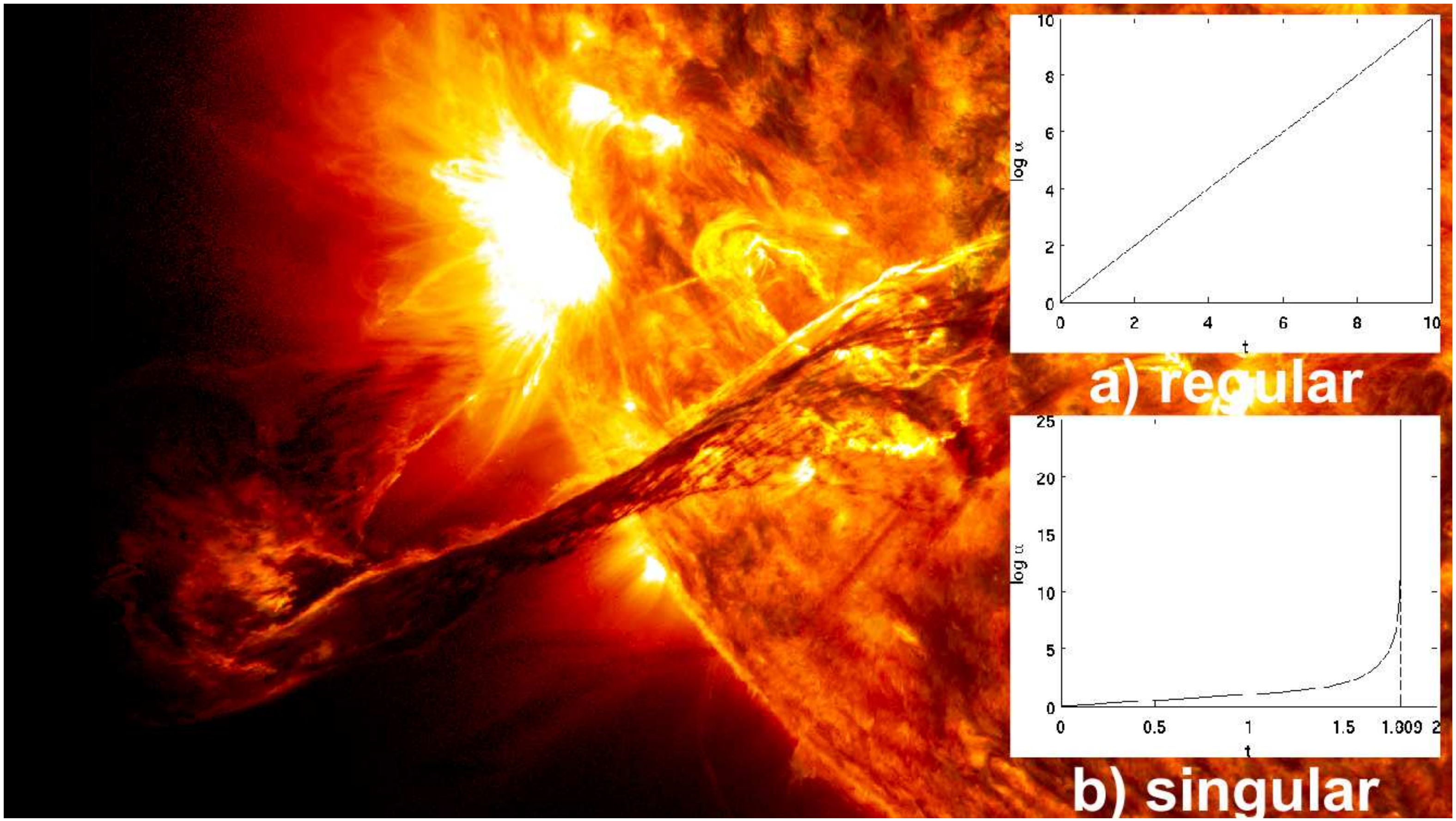
$$h(t) \approx \left(h_0 \cosh(at) + \frac{\dot{h}_0}{a} \sinh(at) \right) \times$$

$$\left[1 - \frac{d_i^2}{16a^2} \left(h_0 + \frac{\dot{h}_0}{a} \right)^2 \exp(2at) \right]^{-1}$$

Singularity time, $h(t_s) = \infty$:

$$t_s = \frac{1}{2a} \ln \left[\frac{16a^2}{d_i^2} \left(h_0 + \frac{\dot{h}_0}{a} \right)^{-2} \right]$$

Solar flares as finite-time singularities?



(SDO EUV image of an eruption on 31/08/2012)

An estimate for the solar flare onset time

$$L = 10^{9.5}\ \mathrm{cm}, \quad T = 10^6\ \mathrm{K}, \quad n = 10^9\ \mathrm{cm}^{-3},$$

$$B = 10^2\ \mathrm{G}, \quad v_A = \frac{B}{\sqrt{4\pi m_p n}} \simeq 10^9\ \mathrm{cm\ s}^{-1}$$

$$d_i = \frac{c}{L\omega_{pi}} \simeq 10^{-6.5}$$

$$a \simeq h_0 \simeq 1 \Longrightarrow t_s \simeq \frac{L}{v_A} \ln \frac{1}{d_i} \simeq 30\,\mathrm{s}$$

Summary

- Observational data and theoretical models strongly suggest that reconnecting current sheets, separating the interacting magnetic fluxes in the solar atmosphere, play the key role in the dynamics and energetics of solar flares.
- The current sheet formation can be modelled as the development of a singularity in the solution for the electric current density at a magnetic neutral line. A finite-time collapse to the current sheet can occur in a weakly collisional plasma, described by Hall magnetohydrodynamics.
- Predictions made using the exact self-similar solutions may have important implications for magnetic reconnection in the laboratory and space plasmas.