A User's Guide to Nonlinear Force-free Fields

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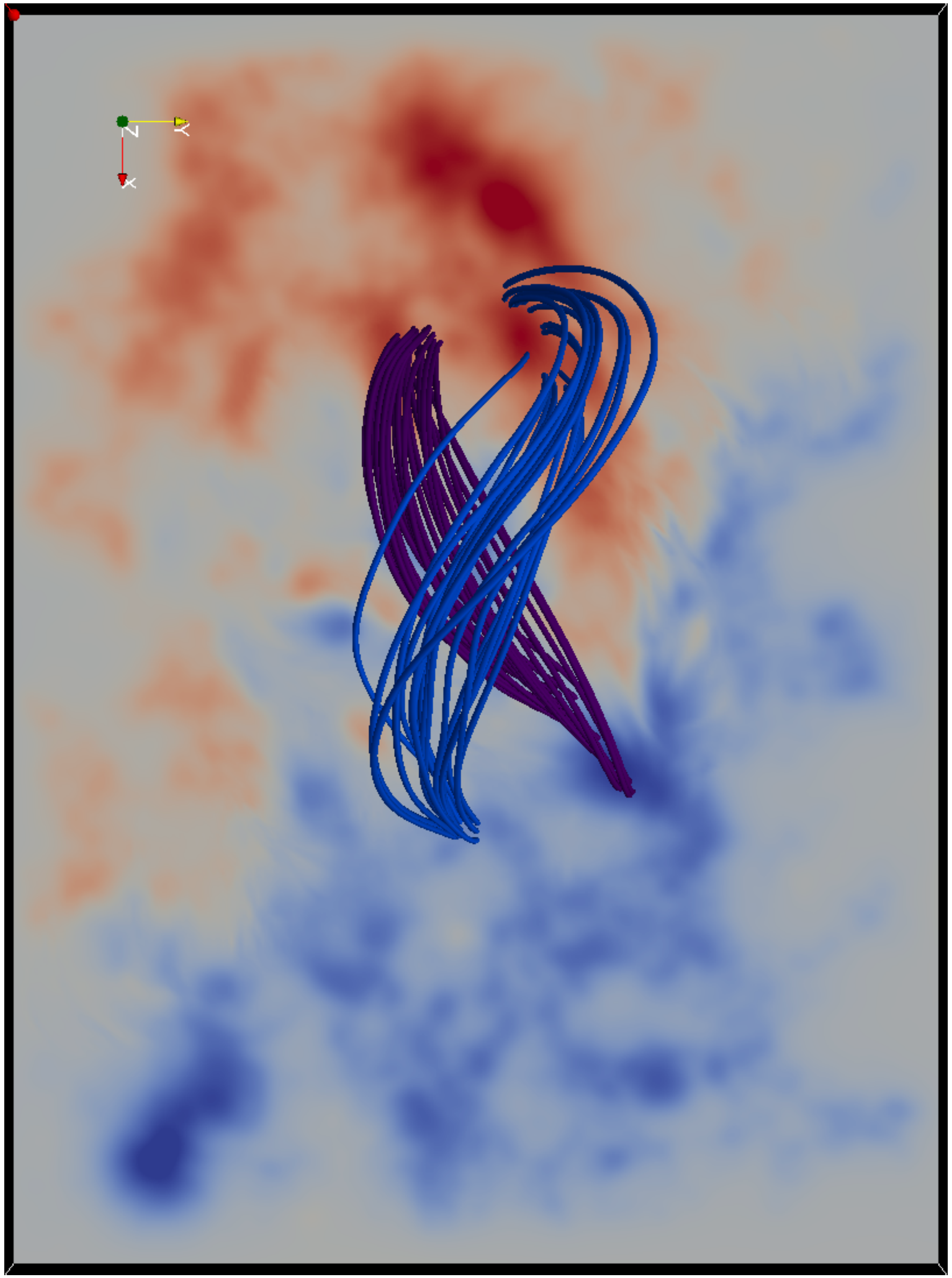

AR10978 on 12 Dec 2007 (DeRosa et al. ApJ 811 107 2015)

Overview

Background

The utility of NLFFF modeling

NLFFF modeling

The data

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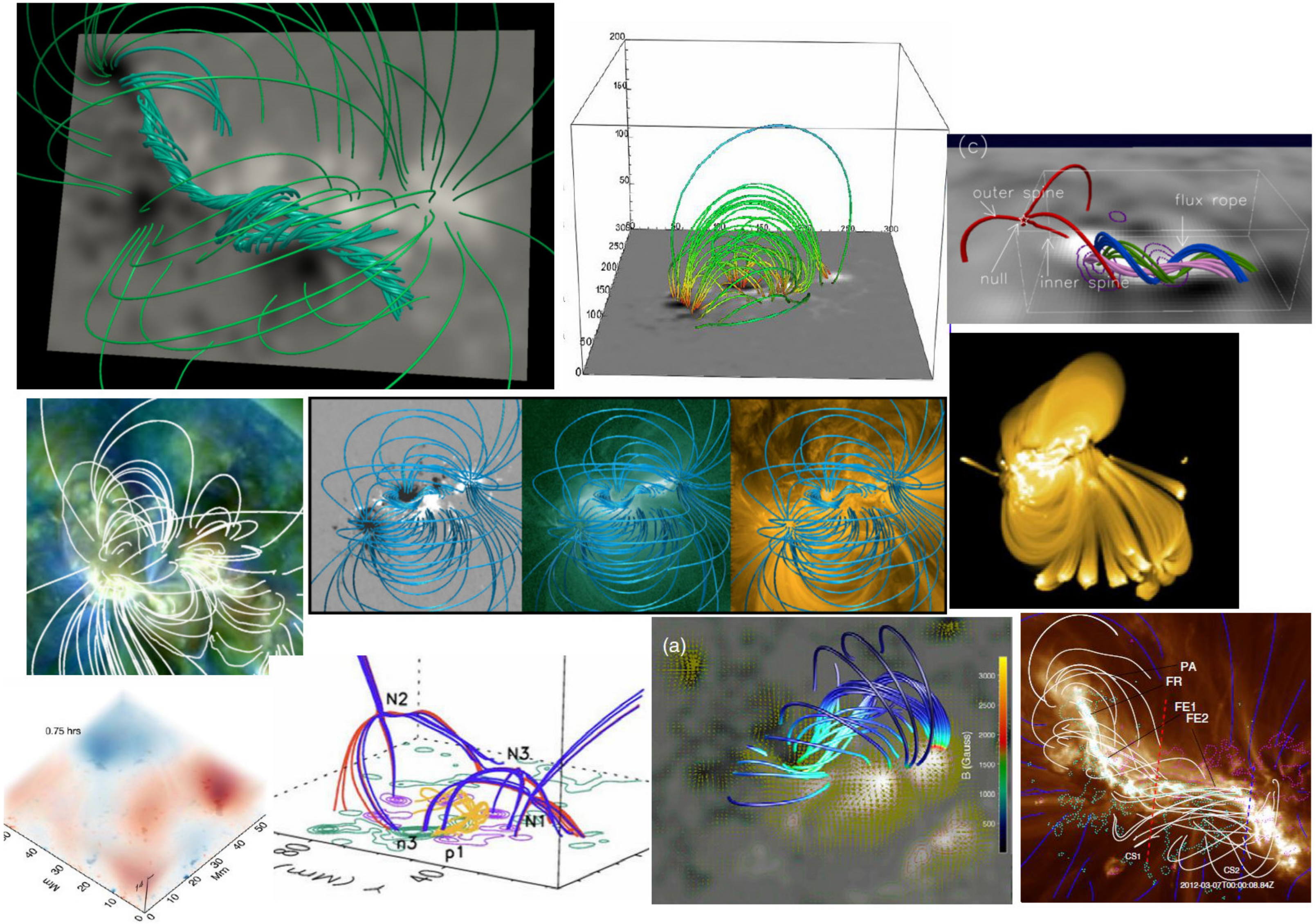
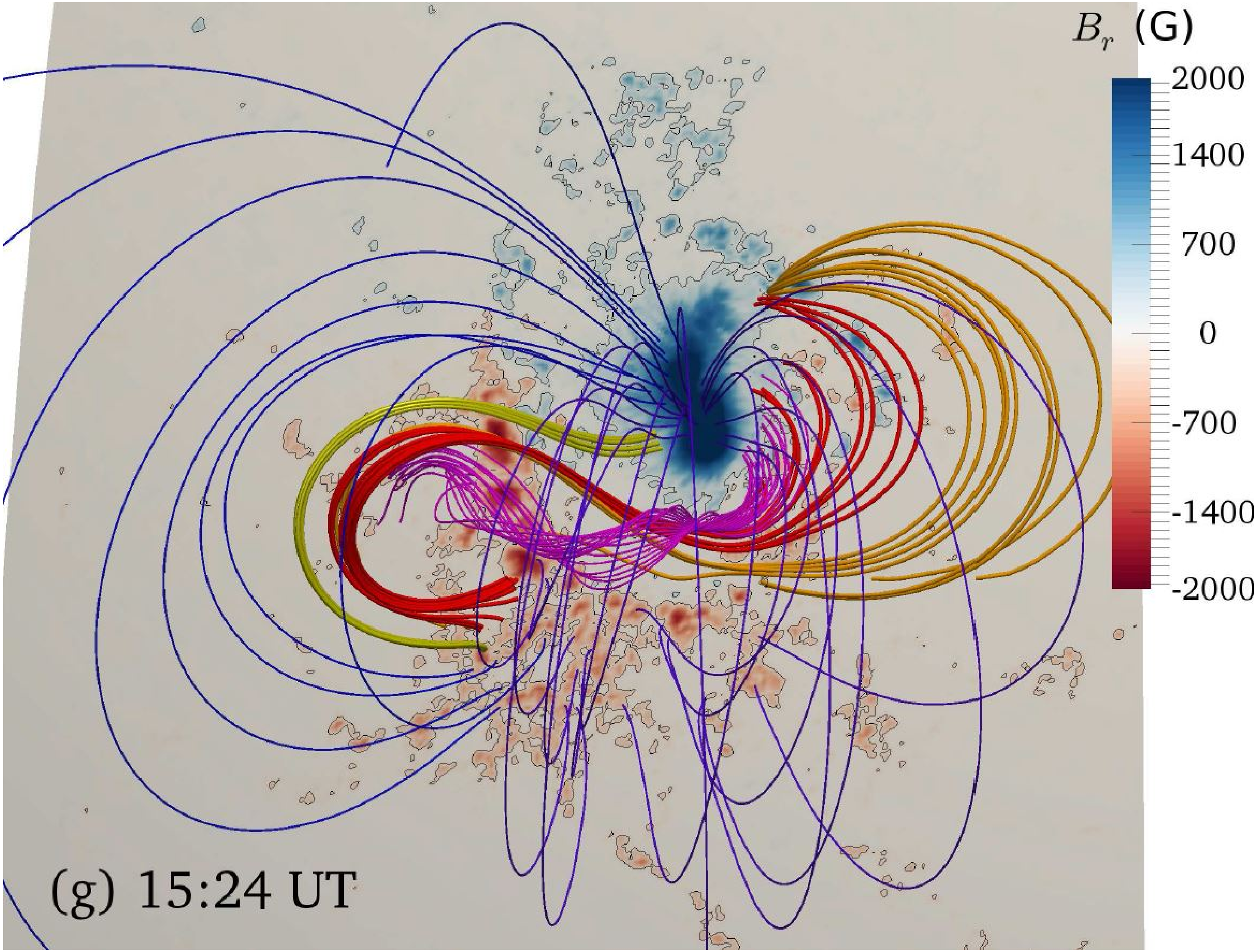
Summary

Background: The utility of NLFFF modeling

- Sunspot magnetic fields power solar flares, eruptions (CMEs)
  - we would like to better understand the source regions
- The coronal field is not amenable to direct measurement
  - vector magnetograms provide values in the low atmosphere
- Model fields may be constructed from the available data
  - a process referred to as extrapolation or reconstruction
- The nonlinear force-free field (NLFFF) model is often used
  - the actual coronal field may be approximately force-free (e.g. Metcalf et al. 1995)
- NLFFF extrapolation is popular
  - the model requires only field values as boundary conditions
  - the model includes electric currents and hence has free energy
    - the solutions may quantify the energy available for activity
  - NLFFF modelling is enabling many studies (e.g. ADS suggests ≈ 50 refereed publications in 2014 using NLFFF extrapolation)



Top row (L to R): Chintzoglou et al. (2015); Moraitis et al. (2014); Yang et al. (2015). Middle row: Tadesse et al. (2015); Inoue et al. (2014); Cheung et al. (2015). Bottom row: Chitta et al. (2014); Mandrini et al. (2014); Cheng et al. (2014); Wang et al. (2014).



AR12158 on 10 September 2014 (courtesy S.A. Gilchrist)

NLFFF modeling: The data

- Zeeman effect imprints **B** on photospheric lines (del Toro Iniesta 2003)
  - Stokes polarisation profiles $I(\lambda)$, $Q(\lambda)$, $U(\lambda)$, $V(\lambda)$ measured
  - Stokes inversion is the process of inferring magnetic field
  - an inference rather than a direct measurement/observation
- 180 degree ambiguity in $B_\perp$ must be resolved
  (Metcalf 1994; Metcalf et al. 2006; Leka et al. 2009)
- Vector magnetogram: photospheric map of $\mathbf{B} = (B_x, B_y, B_z)$
  - local heliocentric co-ordinates (z is local radial direction)
- New space-based instruments
  - Hinode satellite
    - Solar Optical Telescope Spectro-Polarimeter (SOT/SP)
      (Tsuneta et al. 2008)
  - Solar Dynamics Observatory satellite
    - Helioseismic & Magnetic Imager (SDO/HMI)
      (Schou et al. 2012)
- The data provide BCs for NLFFF modeling

NLFFF modeling: The model and the BCs

- Force-free model for coronal magnetic field $\mathbf{B}$:
  (Wiegelmann & Sakurai, Living Reviews of Solar Physics, September 2012)

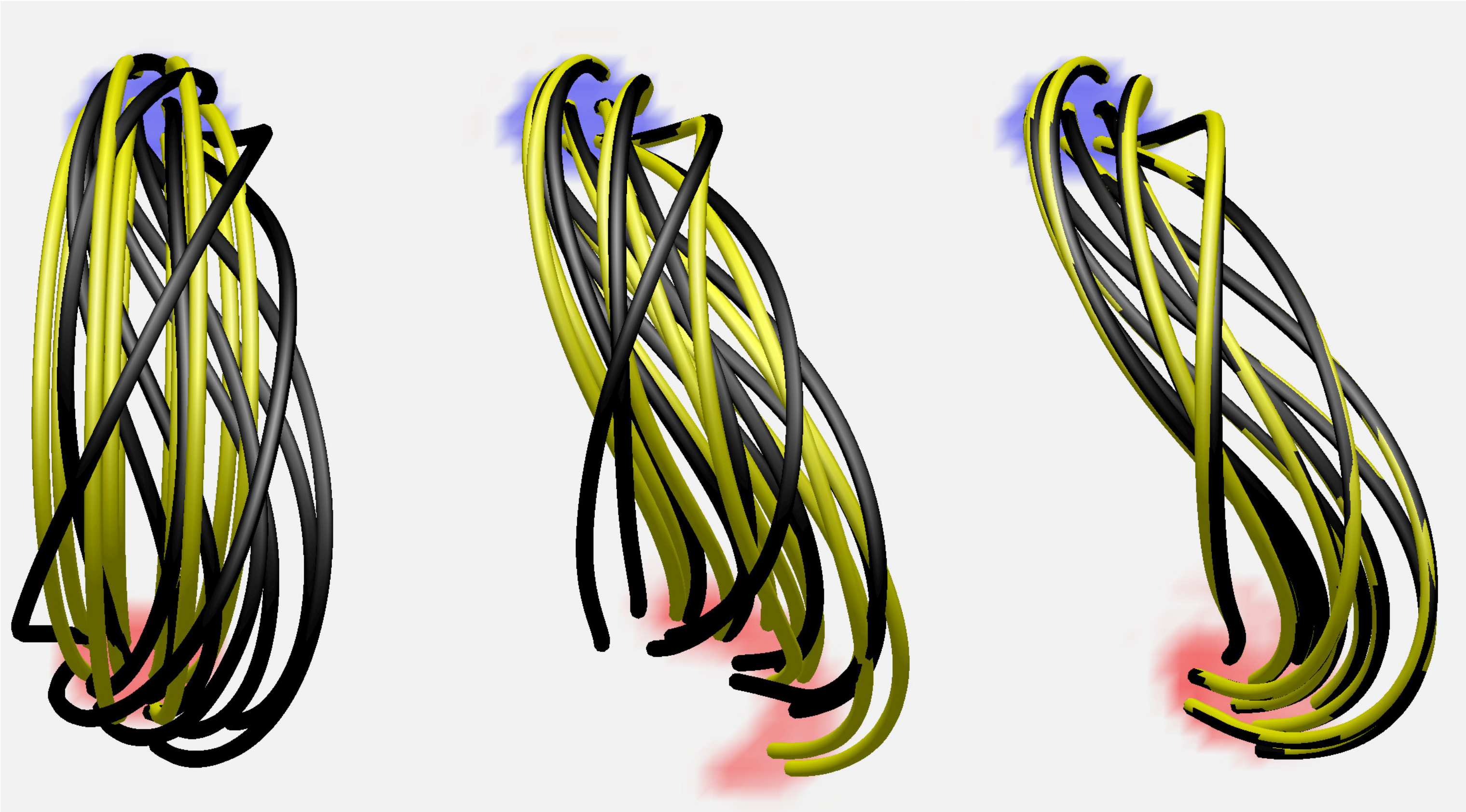
$$\mathbf{J} \times \mathbf{B} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

  - electric current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$
- Writing $\mathbf{J} = \alpha \mathbf{B}/\mu_0$ ($\mathbf{J}$ is parallel to $\mathbf{B}$):

$$\mathbf{B} \cdot \nabla \alpha = 0 \quad \text{and} \quad \nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{2}$$

  - α is the force-free parameter
- Boundary conditions in a half space: (Grad & Rubin 1958)
  - 1. B_z values at $z = 0$
  - 2. α values at $z = 0$ over one polarity of B_z (labelled P, N)
    - equivalently: 2. $J_z = \alpha B_z$ over one polarity of B_z
- Some force-free methods use $\mathbf{B}$ over both P and N as BCs
  - this defines α (or J_z) over both polarities
  - it is formally an over-prescription

- Methods of solution of Eqs. (2) are iterative
  - Grad-Rubin iteration (Grad & Rubin 1958; Amari et al. 2006; Wheatland 2007)
  - optimization (Wheatland, Sturrock & Roumeliotis 2000; Wiegelmann 2008)
  - magnetofrictional (Chodura & Schlúter 1981; Valori, Kliem, and Keppens 2005)
- Grad-Rubin method
  - 1. currents are run along field lines subject to BCs on α
  - 2. the currents perturb the field; return to 1.



Three stages in a G-R iteration showing field lines (black) and current streamlines (yellow).

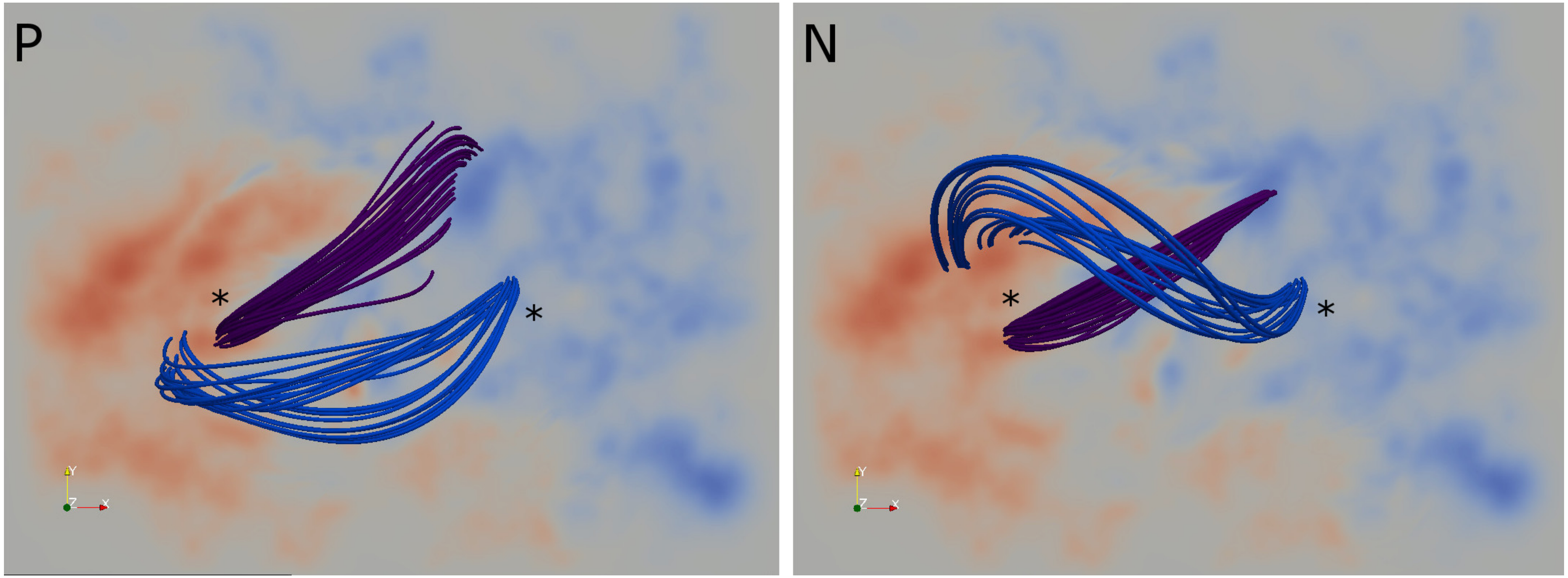
NLFFF modeling: The problem of inconsistency

- Magnetogram data are inconsistent with the force-free model
  - necessary conditions for a force-free field are not met (Molodenskii 1969)
- Origins of the inconsistency of the data and the model:
  - errors in measurements and field inference (e.g. Leka et al. 2009)
  - non-magnetic forces at the dense photosphere (Metcalf et al. 1995)
- Effect of inconsistency on NLFFF solutions (Schrijver et al. 2006; Metcalf et al 2008; Schrijver et al. 2008; DeRosa et al. 2009, DeRosa et al. 2015)
  - results produced by NLFFF codes may not agree:
    - with the NLFFF model, for a given solution method
    - with other methods of solution, for the same BCs
    - with equally valid results for the same method
  - different results may have:
    - different energies and free energies
    - different field line structures

NLFFF modeling: The effects of inconsistency

Grad-Rubin methods

- Vector magnetograms provide α values over P and N
  - the data provide two sets of boundary conditions
  - two different solutions (the P and N solutions) are implied
- The iteration sequence may not converge (for P and/or N)
  - Convergence may be achieved by:
    - smoothing of B_x and B_y values or α values (e.g. Canou et al. 2009)
    - censoring α values in weak field regions (e.g. Wheatland & Leka 2011)

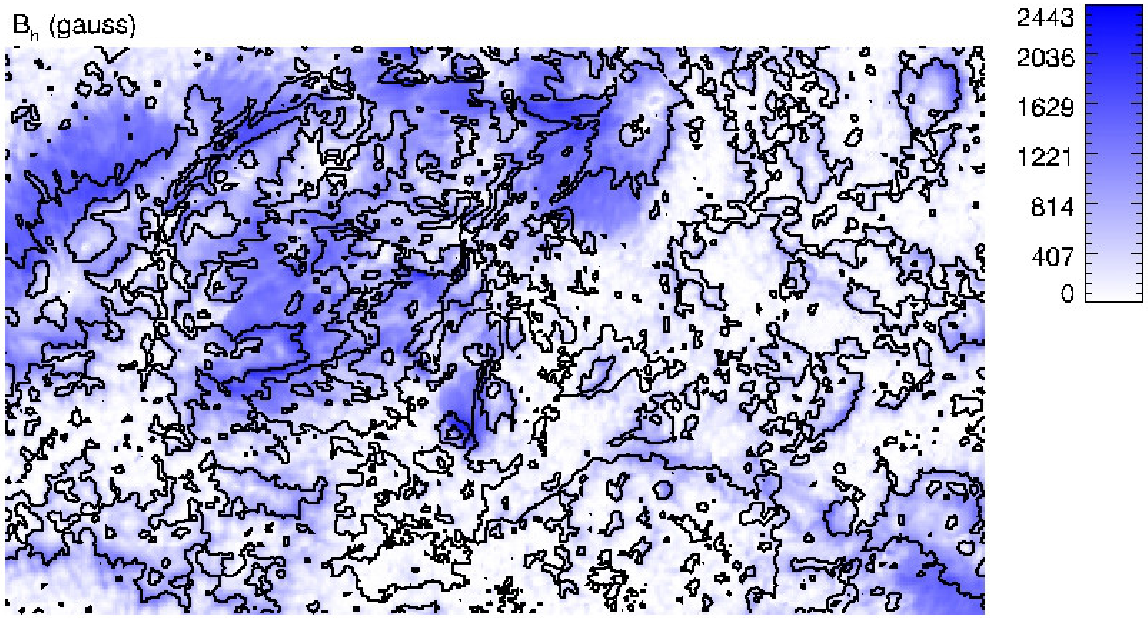


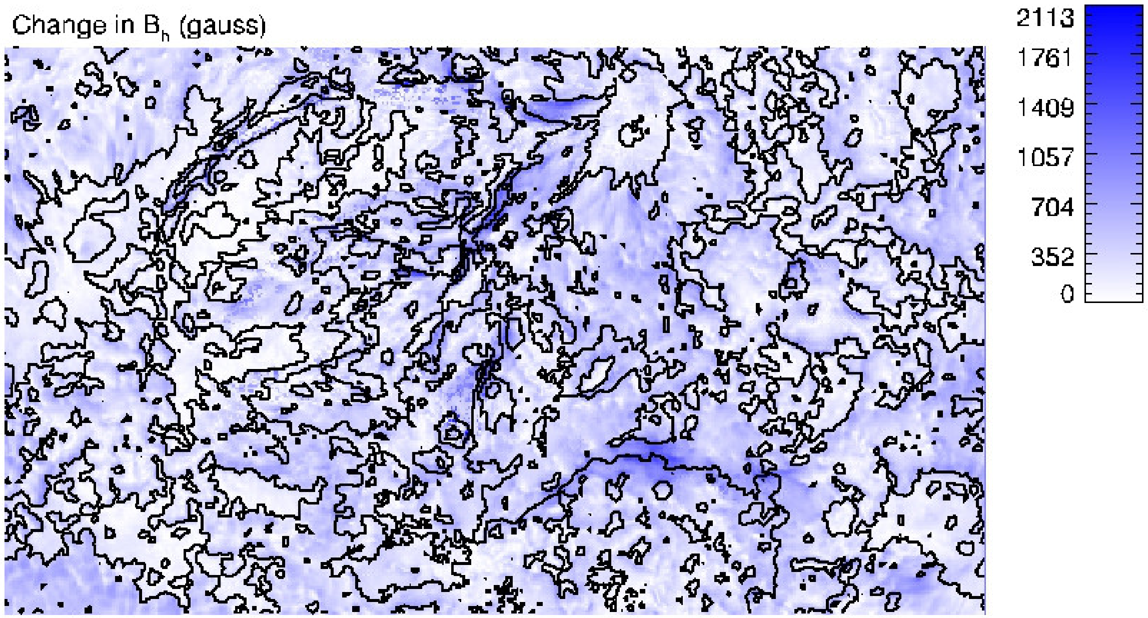
P and N solutions for AR10978 on 12 Dec 2007 – CFIT bin 4 solution as described in DeRosa et al. (2015)

Optimization

- Optimization is a popular method
  (Wheatland, Sturrock & Roumeliotis 2000; Wiegelmann 2008)
  - the method uses values of **B** over P and N as BCs
  - so there is only one result...
  - ...but for inconsistent BCs it is not an accurate NLFFF solution
- The result may have $\mathbf{J} \times \mathbf{B} \neq 0$ and/or $\nabla \cdot \mathbf{B} \neq 0$
  - optimization explicitly involves departure from $\nabla \cdot \mathbf{B} = 0$
- Preprocessing is often applied
  (Wiegelmann et al. 2006; Wiegelmann & Inhester 2010)
  - the BCs are altered to meet necessary conditions for the model
  - this procedure may improve the quality of solutions
  - but preprocessed BCs are still inconsistent with the model
    (DeRosa et al. 2009)

- Quite generally, methods change the BCs substantially
  - Values of $B_h = \left(B_x^2 + B_y^2\right)^{1/2}$ and ΔB_h for a G-R calculation:





AR10978 on 12 Dec 2007 – CFIT bin 2 P solution (DeRosa et al. 2015)

NLFFF modeling: How accurate is my solution?

- There are two (related) questions
  - Q1 – Does it represent what is on the Sun?
  - Q2 – Is it can accurate solution to the model?
- Approaches for Q1 may include:
  - comparison of field line traces and EUV/X-ray images
  - comparison with results from other methods and models
  - consideration of the changes in BCs required in modeling
- Common metrics for Q2 include: (Wheatland et al. 2000)
  - the pointwise average of $|\nabla \cdot \mathbf{B}_i|$ over grid points i
  - the weighted average angle between $\mathbf{J}_i$ and $\mathbf{B}_i$ over the grid
- However these ‘answers’ to Q2 are hard to interpret
  - the significance of a value depends on how a solution is used

Q2: If you are using solutions to estimate energy

- Check the non-solenoidal contributions to energy: (Valori et al. 2013)
  - Perform the Helmholtz decomposition:

$$E = E_{\mathrm{p,s}} + E_{\mathrm{p,ns}} + E_{\mathrm{J,s}} + E_{\mathrm{J,ns}} + E_{\mathrm{mix}} \tag{3}$$

$$E_{\mathrm{p,s}} = \frac{1}{2\mu_0}\int \mathbf{B}_{\mathrm{p,s}}^2\, dV \qquad E_{\mathrm{p,ns}} = \frac{1}{2\mu_0}\int \mathbf{B}_{\mathrm{p,ns}}^2\, dV \tag{4}$$

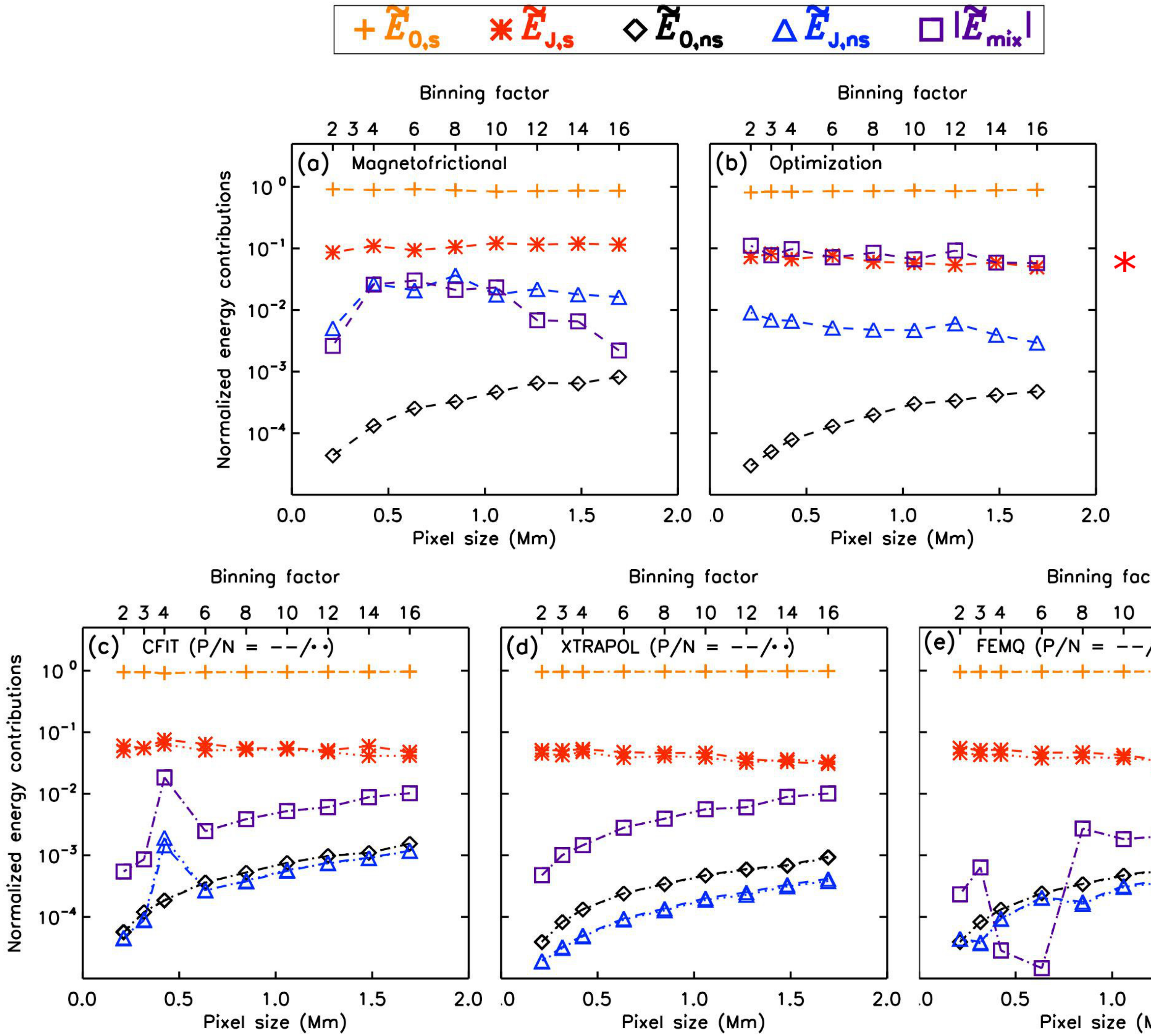
$$E_{\mathrm{J,s}} = \frac{1}{2\mu_0}\int \mathbf{B}_{\mathrm{J,s}}^2\, dV \qquad E_{\mathrm{J,ns}} = \frac{1}{2\mu_0}\int \mathbf{B}_{\mathrm{J,ns}}^2\, dV \tag{5}$$

$$\begin{aligned} E_{\mathrm{mix}} = \frac{1}{\mu_0}\int & (\mathbf{B}_{\mathrm{p,s}}\cdot\mathbf{B}_{\mathrm{p,ns}} + \mathbf{B}_{\mathrm{J,s}}\cdot\mathbf{B}_{\mathrm{J,ns}} + \mathbf{B}_{\mathrm{p,s}}\cdot\mathbf{B}_{\mathrm{J,ns}} \\ & + \mathbf{B}_{\mathrm{J,s}}\cdot\mathbf{B}_{\mathrm{p,ns}} + \mathbf{B}_{\mathrm{p,ns}}\cdot\mathbf{B}_{\mathrm{J,ns}} + \mathbf{B}_{\mathrm{p,s}}\cdot\mathbf{B}_{\mathrm{J,s}})\, dV \end{aligned} \tag{6}$$

    - where p denotes potential and J denotes current-carrying
    - and s denotes solenoidal ($\nabla \cdot \mathbf{B} = 0$) and ns denotes non-solenoidal
  - For a solenoidal field $E = E_p + E_J$ with $E_p = E_{\mathrm{p,s}}$, $E_J = E_{\mathrm{J,s}}$ and

$$E_{\mathrm{p,ns}} = E_{\mathrm{J,ns}} = E_{\mathrm{mix}} = 0 \tag{7}$$

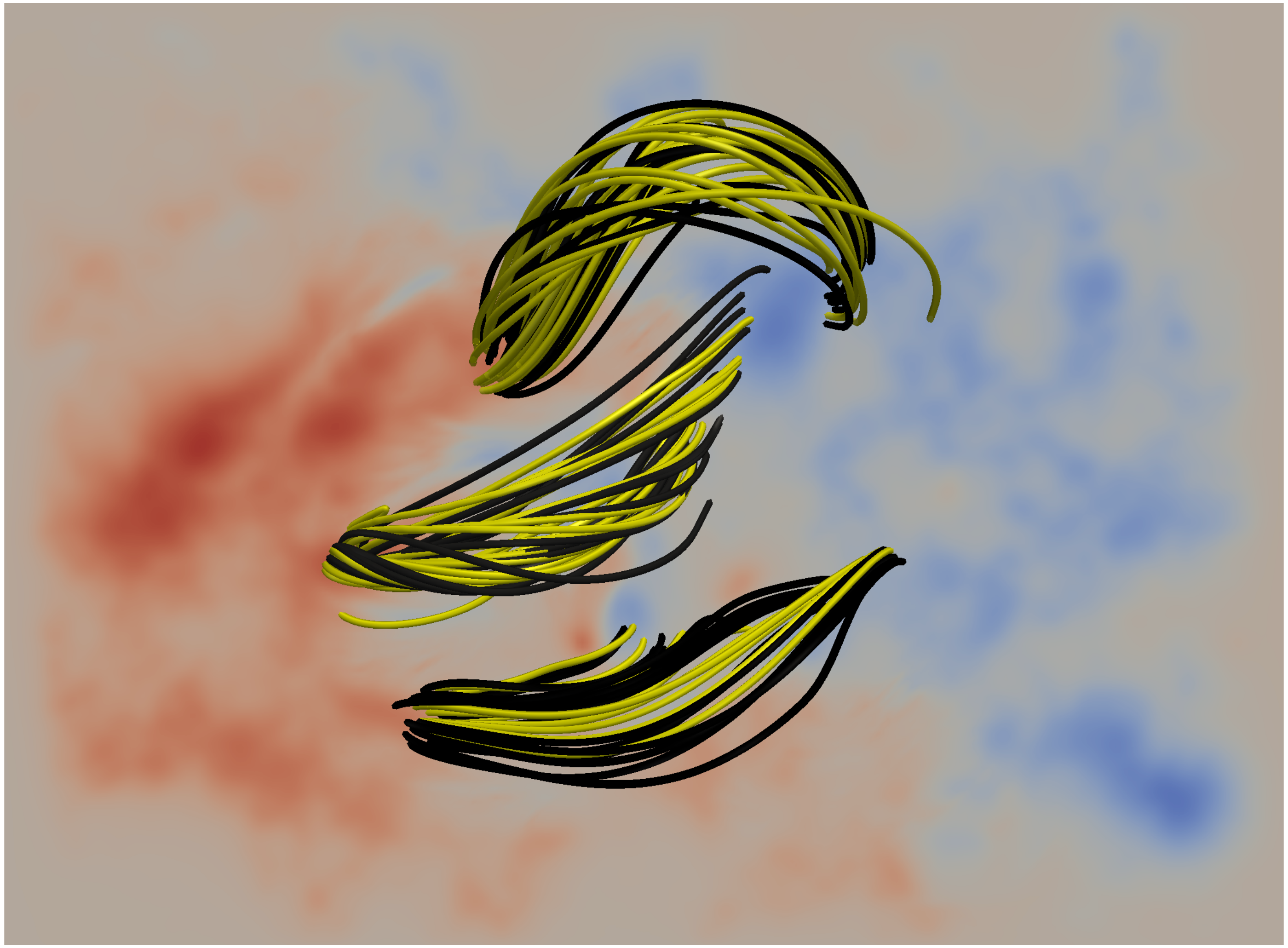
  - check that $E_{\mathrm{p,ns}}$, $E_{\mathrm{J,ns}}$ and $|E_{\mathrm{mix}}|$ are small compared with E_{free}
- Optimization/magnetofrictional calculations may fail the test (DeRosa et al. 2015)



Helmholtz decomposition of field energy for NLFFF calculations for AR10978 on 12 Dec 2007 (DeRosa et al. 2015).

Q2: If you are using solutions to identify specific field structures

- Trace field lines and corresponding current streamlines
  - they should agree if the structure is force-free
  - this is quite a stringent test
  - an example is shown below for a G-R calculation



CFIT bin 4 P solution for AR10978: field lines (black) and current streamlines (yellow) (DeRosa et al. 2015).

Summary

- Coronal magnetic field modeling is motivated by solar activity
- Vector magnetograms give BCs for coronal field extrapolation
- The nonlinear force-free (NLFFF) model is popular
  - but vector magnetogram data are inconsistent with the model
    - methods may give unreliable results for solar data
- If you are using NLFFF modeling check your results
  - consider the quality of solution to the model
  - consider whether it represents what is on the Sun
- Apply tests appropriate for the questions being asked

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