Stability analysis of two-arc electric current loop in the solar corona

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Project for Solar-Terrestrial Environment Prediction

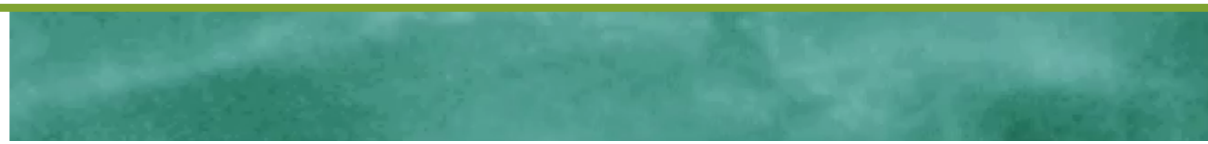
STEL

Introduction (flux rope)

Flux rope → twisted magnetic tube structure

The stability of flux rope ⇔ flare, CME
flare, CME ⇔ substorm, geomagnetic disturbance

→This stability is important issue for space weather.

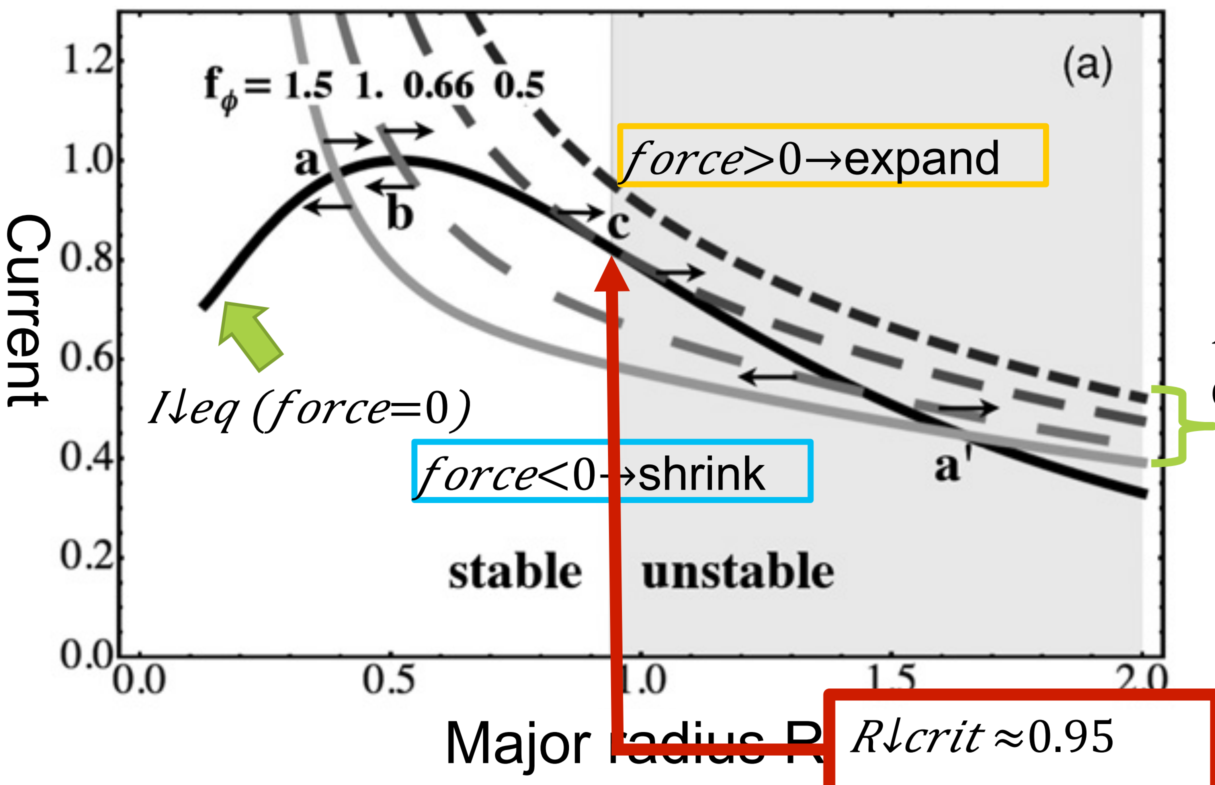


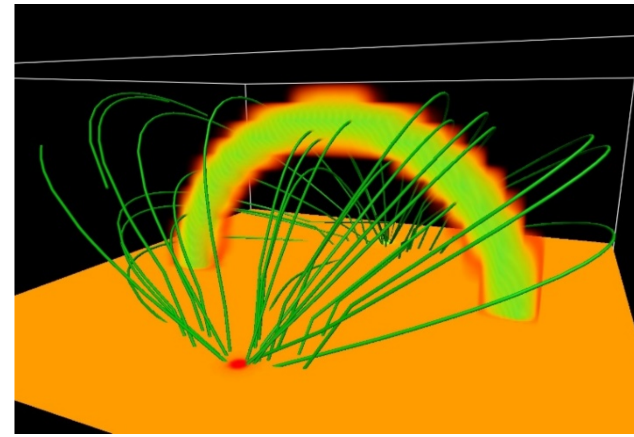
Introduction (torus inst.)

■The axisymmetric current loop model

○Kliem & Toeroek [2006]: Torus Instability

○Demoulin & Aulanier [2010]





green line: magnetic field
torus: axi. current loop

I_{evol} (flux conservation)

critical decay index

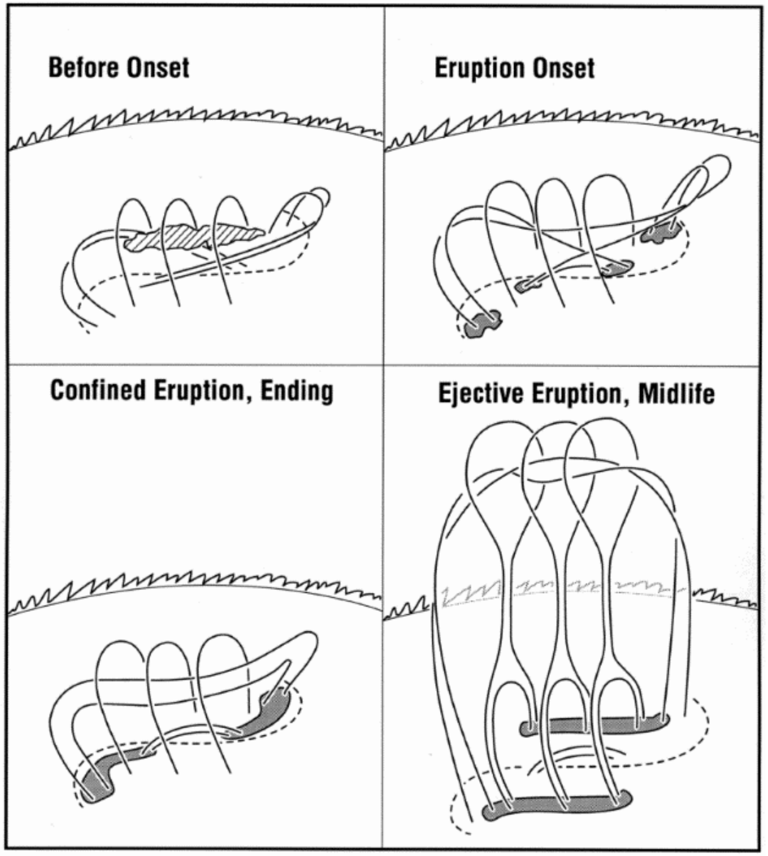
$n > n_{cr} = 3/2 - 1/4c_0 \approx 1.5$

Introduction (tether-cutting)

However, how the unstable state is realized is not elucidated and several scenario were proposed.

■"tether-cutting" reconnection (Moore et al. [2001])

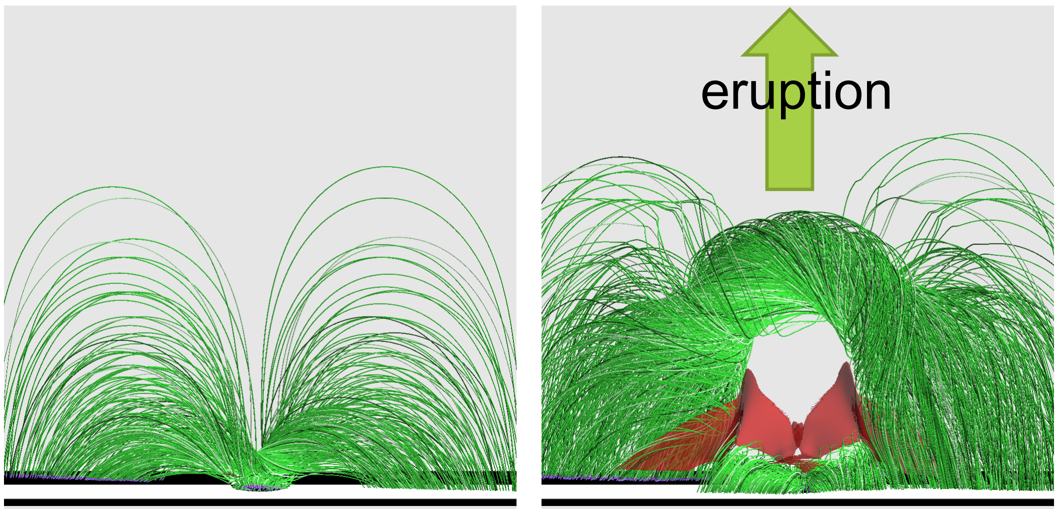
internal reconnection between sheared magnetic loops
↓
destabilization of flux rope
↓
magnetic explosion in eruptive flares and CMEs



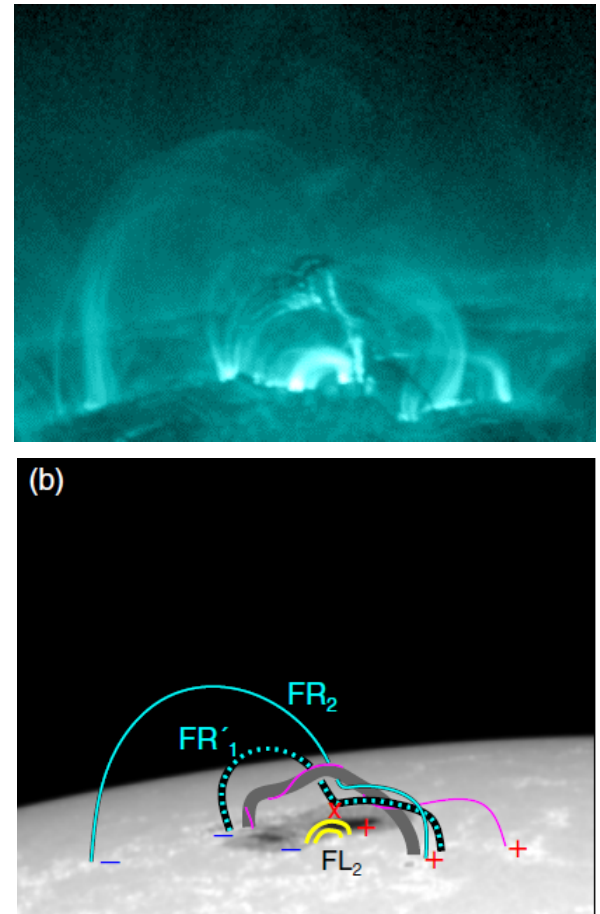
Moore et al. [2001]

Introduction (two-arc)

Some observational and numerical studies show consistent results for tether-cutting:



Kusano et al. [2012]



Chen et al. [2014]

These results show two-arc flux rope in the onset phase of eruption.

However, the <u>stability of two-arc</u> flux rope has <u>not yet been analyzed</u>.

Objectives

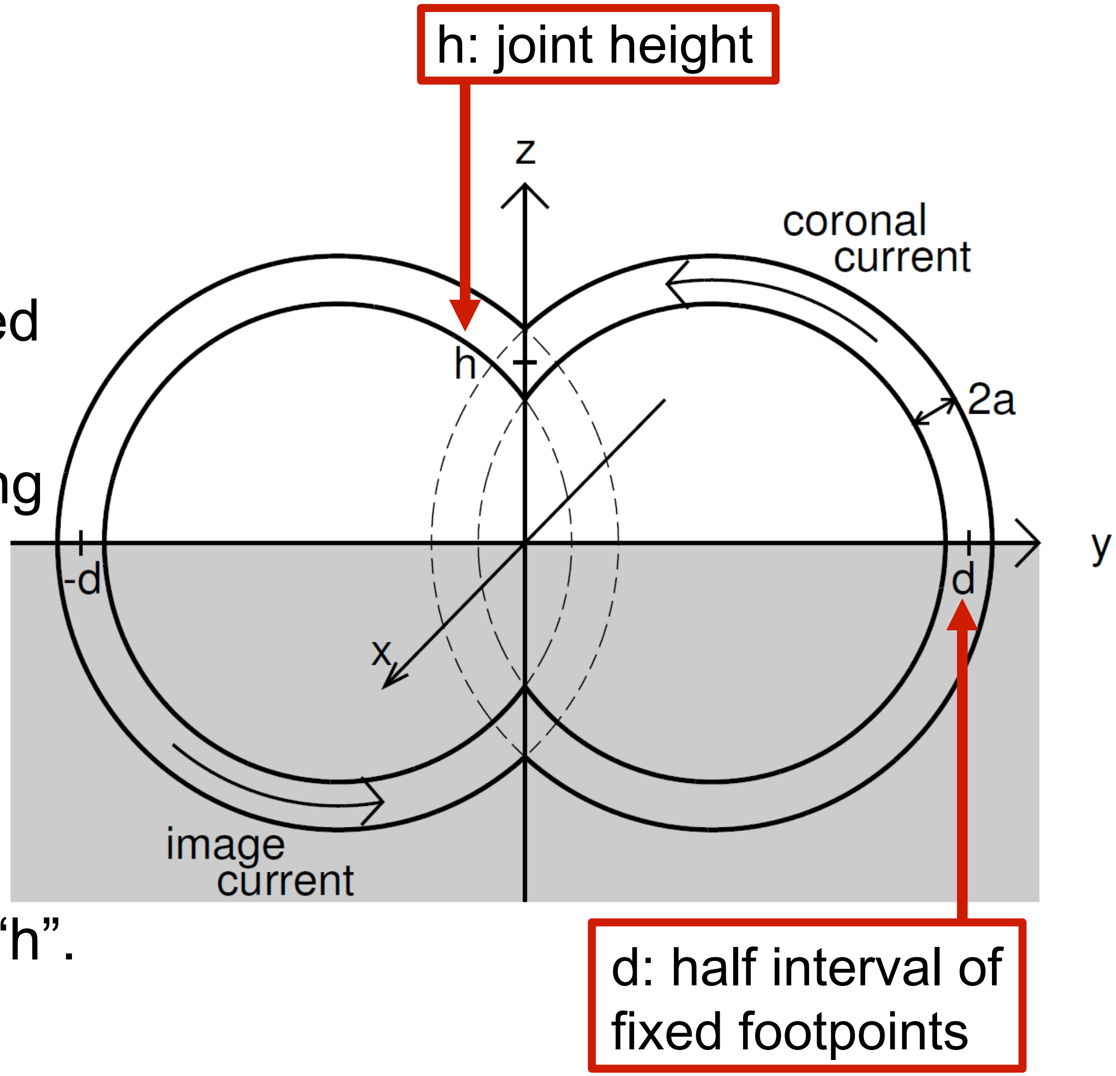
- To reveal the stability and equilibrium condition for the two-arc flux rope in the solar corona
- To develop current loop model to calculate the stability for the two-arc flux rope

As a result, we found that,

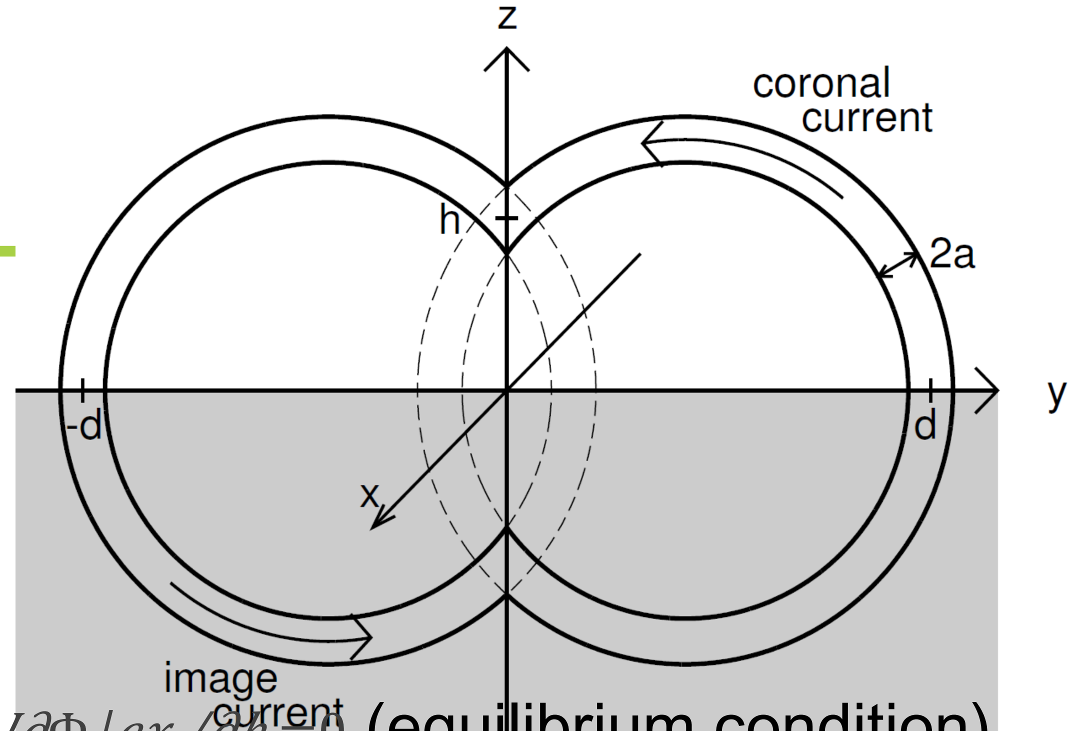
Two-arc flux rope is much easier to destabilize than the axisymmetric torus, and the decay index is not necessarily an adequate criterion for the onset of eruption.

Two-arc model

- ideal MHD
- neglect pressure and gravity (low-β)
- shape: two-arc connected two circular tori
- flow uniform current along inner the loop
- line-tied assumption: fix footpoints “d”
- we can determine the shape uniquely by parametrizing joint height “h”.



Equations



The energy of the loop

$$U = \frac{1}{2} L I^2 + I \Phi_{ex}$$

Generalized force

$$F = \frac{\delta U}{\delta h} = \frac{1}{2} I^2 \frac{\partial L}{\partial h} + I \frac{\partial \Phi_{ex}}{\partial h} = 0$$ (equilibrium condition)

$$\rightarrow I_{eq}(h) = -2 \frac{\partial \Phi_{ex}(h)}{\partial h} \Big/ \frac{\partial L(h)}{\partial h}$$

Magnetic flux through the loop:

$$\Phi_{total} = LI + \Phi_{ex} = const$$ (conservation of magnetic flux)

$$\rightarrow I_{evol}(h) = \frac{1}{L(h)} \left(\Phi_{total} - \Phi_{ex}(h) \right)$$

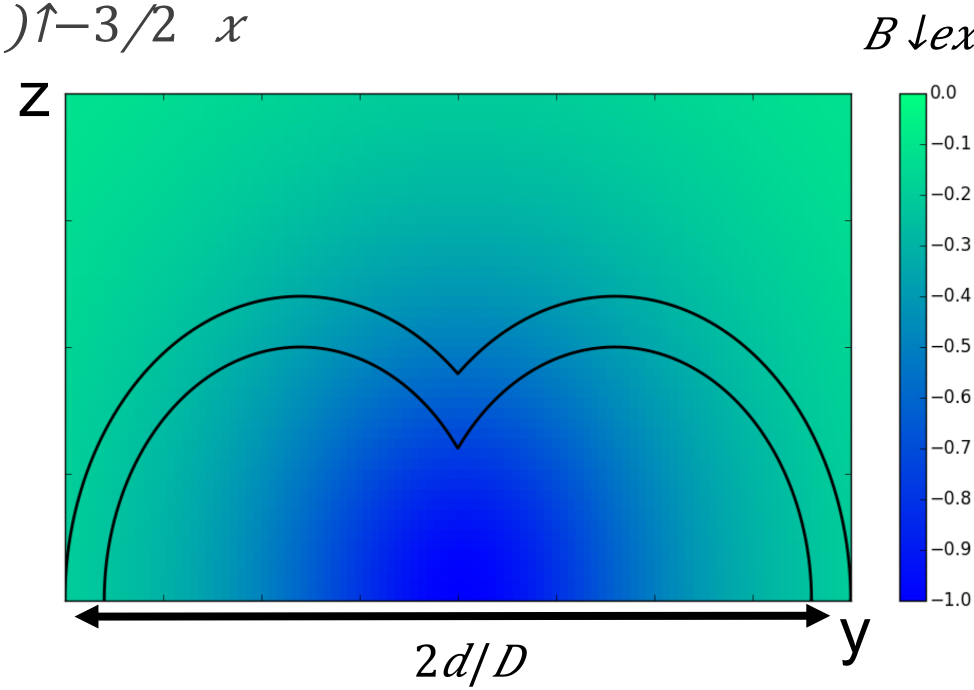
Setting

We calculated I_{eq} and I_{evol} for two-arc loop numerically.

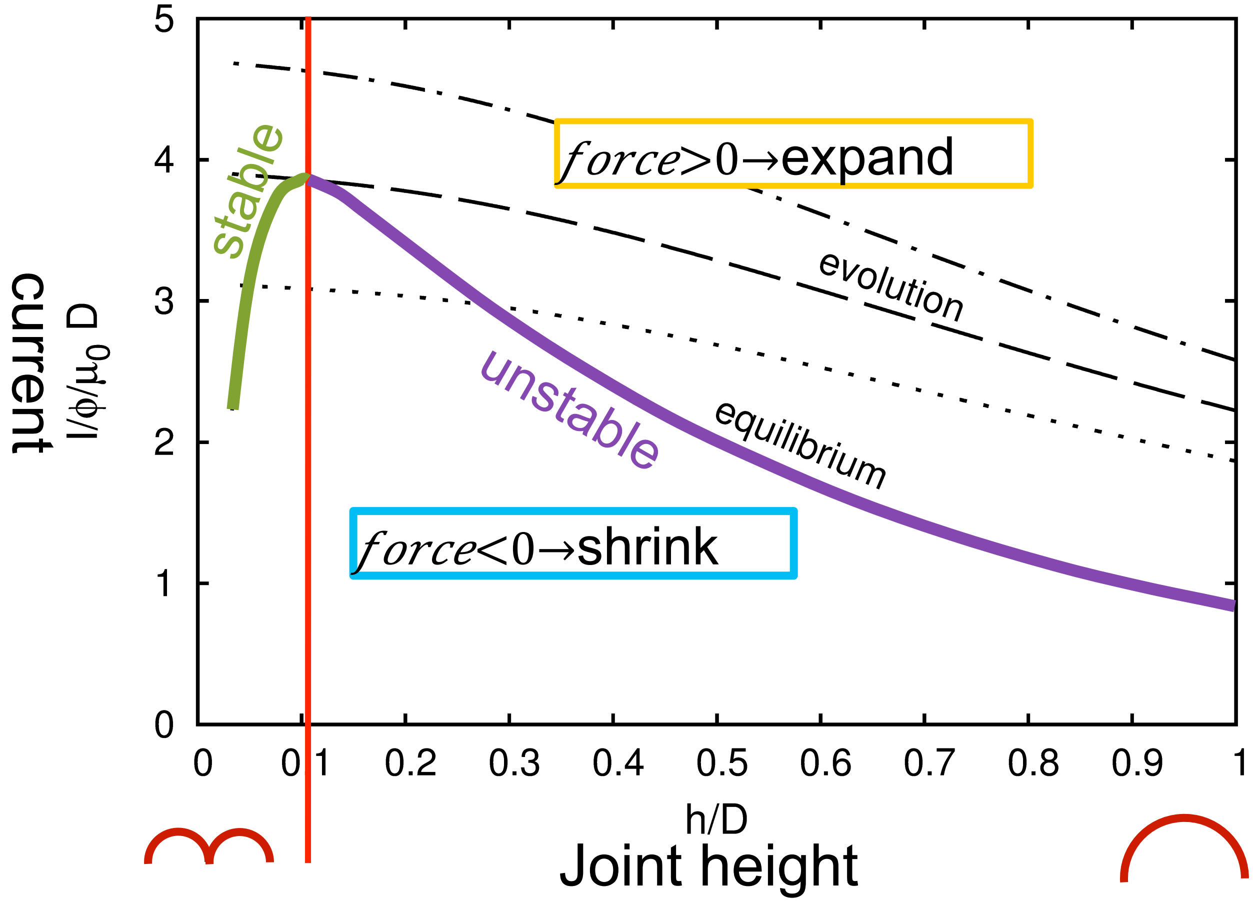
External magnetic field is given by a <u>point source bipole</u> at a <u>point distant D</u> from the polarity inversion line.

$$\vec{B}_{ex} = -4\phi D (y^2 + z^2 + D^2)^{-3/2} \; x$$

Parameter	
Toroidal grid number	4000
Poloidal grid number	5*30
$2a/D$ (width)	0.002
$2d/D$ (interval between footpoints)	2.0



Result (bipole field)



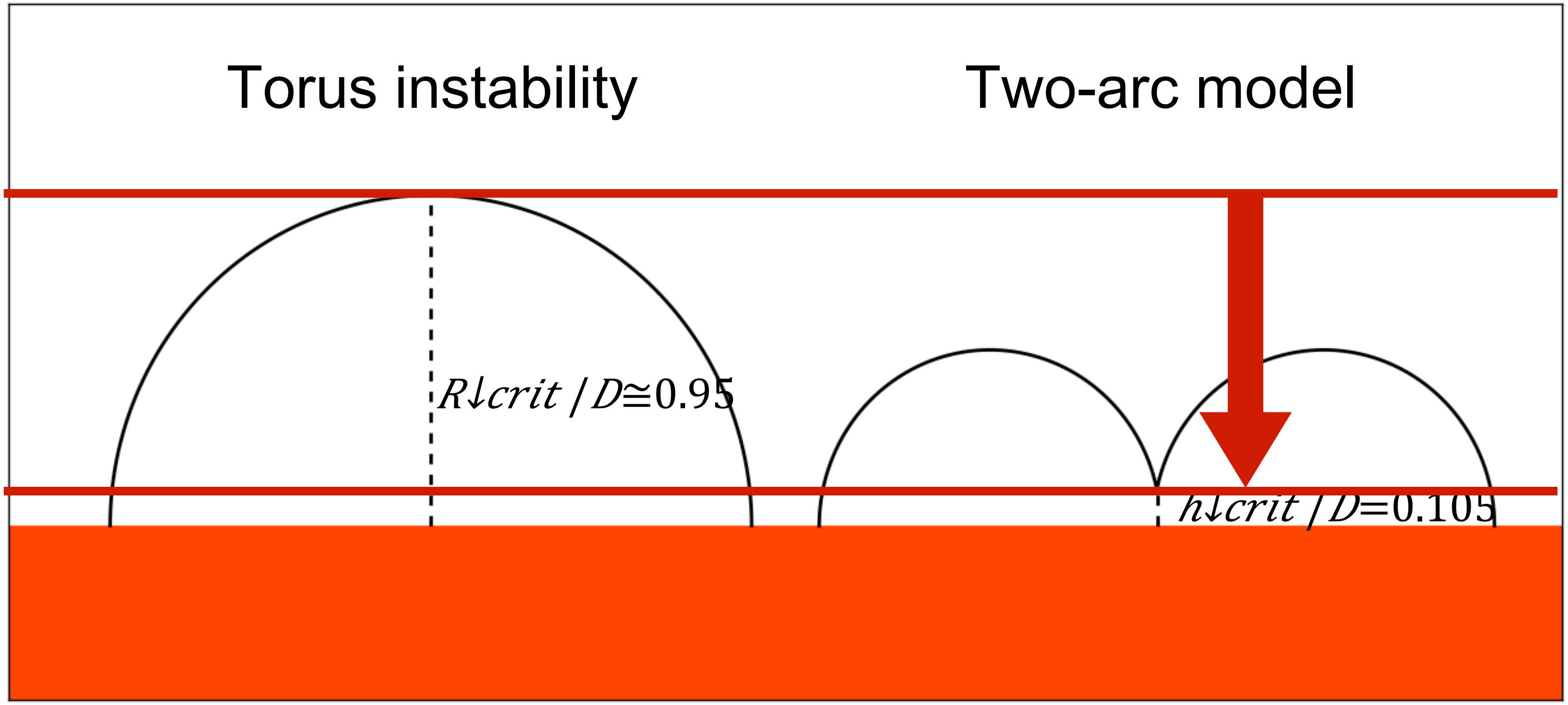
$\rightarrow h_{crit}/D = 0.105$

h: joint height
D: distance of external field source from PIL

$\Phi_{total} = 0.7\Phi_0$
$\Phi_{total} = 1.0003\Phi_0$
$\Phi_{total} = 1.3\Phi_0$

Grid number	4000*5*30
width (2a/D)	0.002
distance between footpoints (2d/D)	2.0

Compare critical height…



critical radius $R_{crit}/D \cong 0.95$

critical height $h_{crit}/D = 0.105$

There is lower critical height for two-arc loop than torus inst.

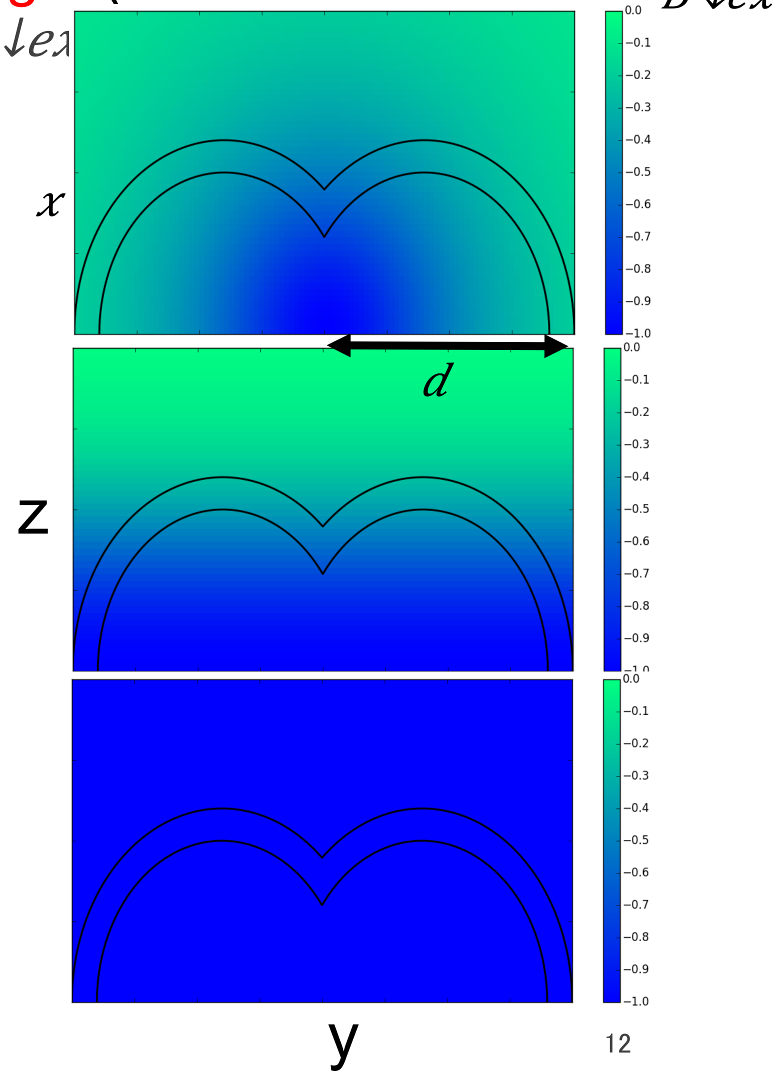
Other parameter case

We also calculate h_{crit} by changing d (the interval b/w footpoints), and for three types of B_{ex}

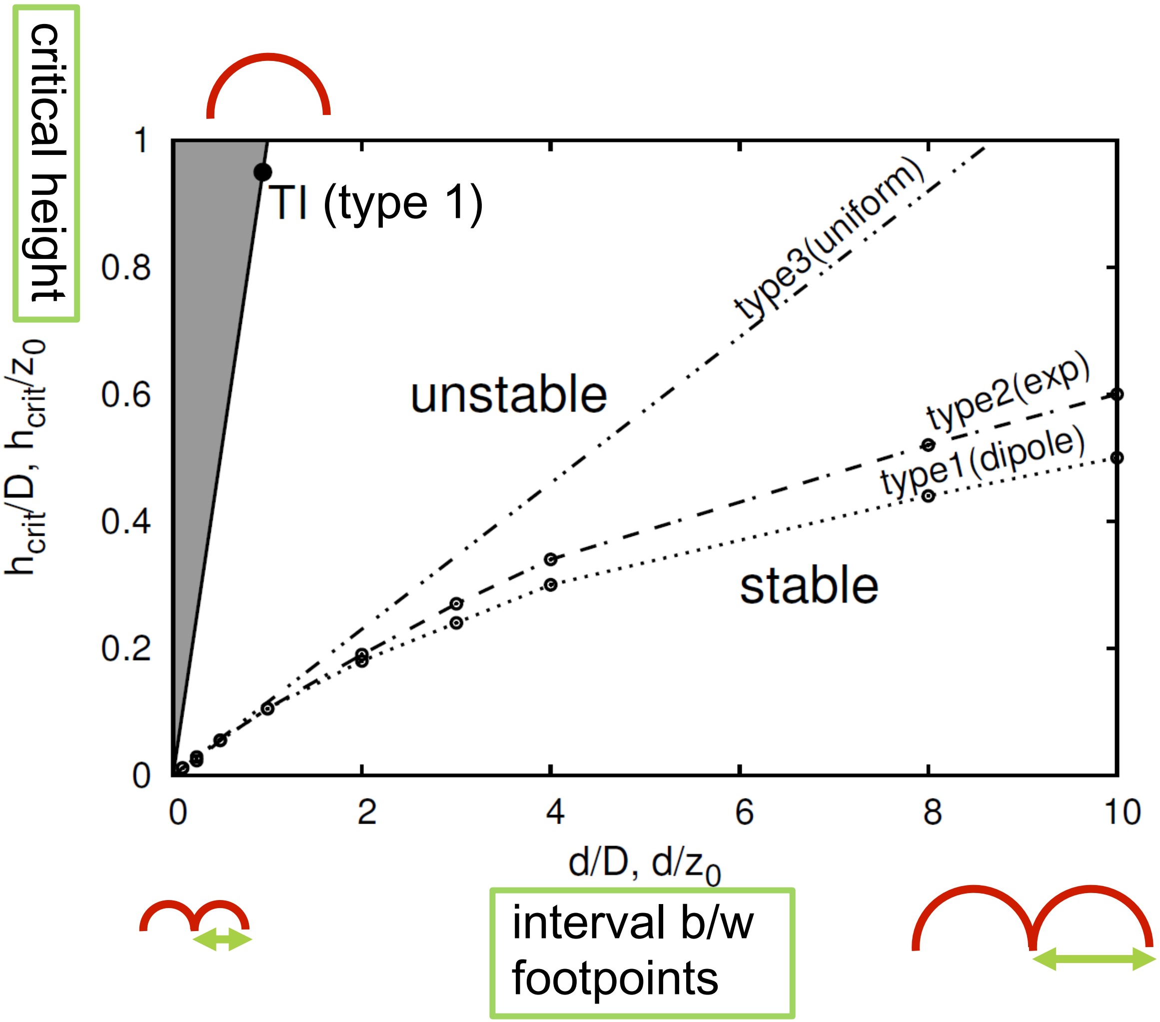
1. point source bipole
$B_{ex} = -4\phi D (y^2 + z^2 + D^2)^{-3/2}\ x$

2. decaying field exp. with altitude
$B_{ex} = -B_0 e^{-|z|/z_0}\ x$

3. uniform field
("zero decay" field)
$B_{ex} = -B_0\ x$



Critical height for three different cases

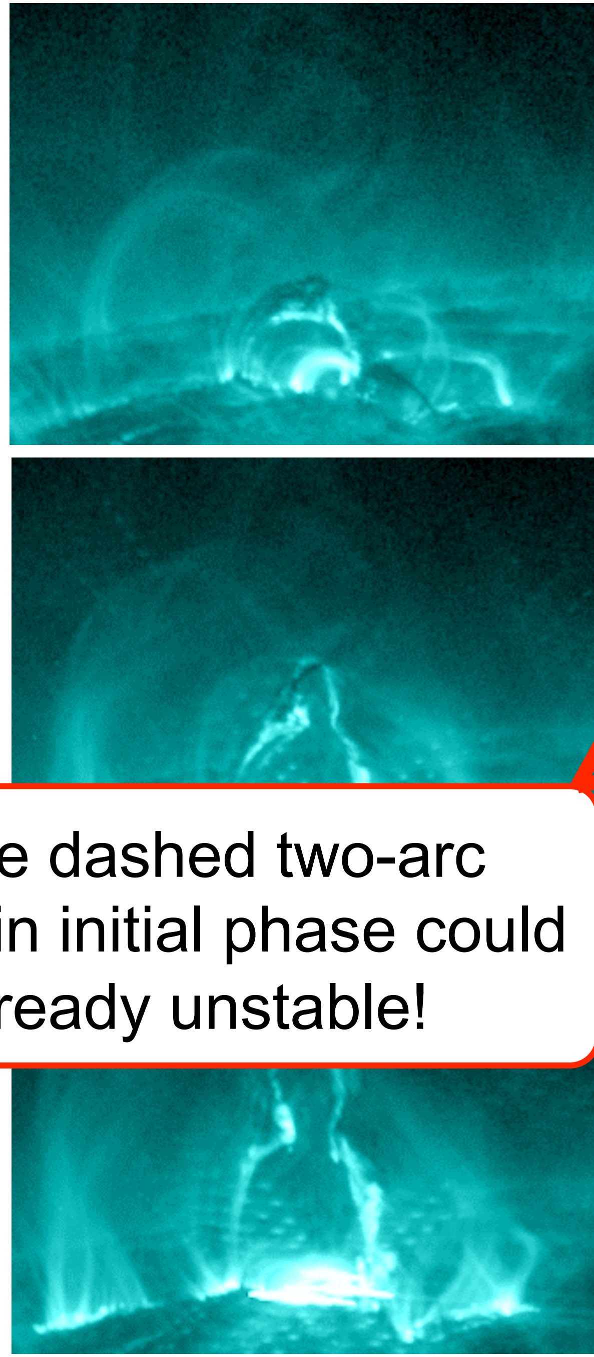


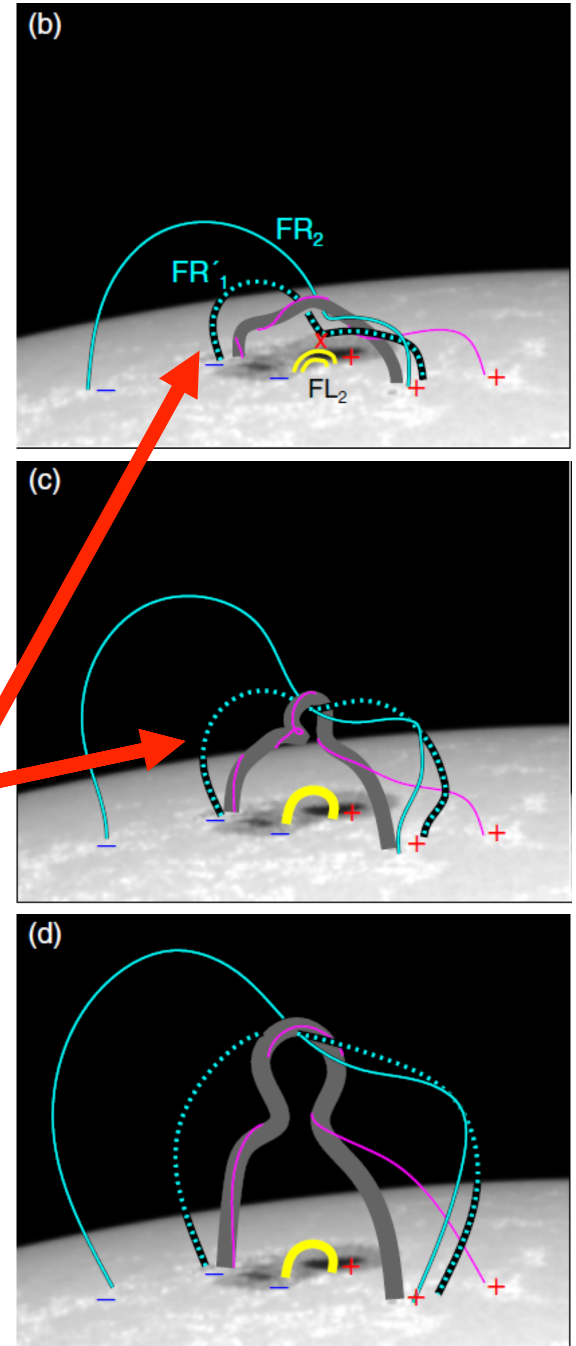
h: joint height
d: interval b/w footpoints
D: distance of external field source from PIL (bipole)
z_0 : scale length for exp. field

○ Two-arc loop can be destabilized even for <u>uniform (zero decay) field</u>.

○The critical joint height is much lower than the footpoint interval d.

Time



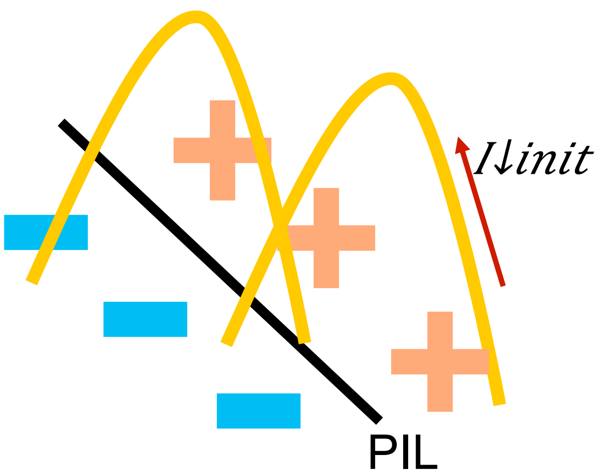


These dashed two-arc loop in initial phase could be already unstable!

Chen et al. [2014]

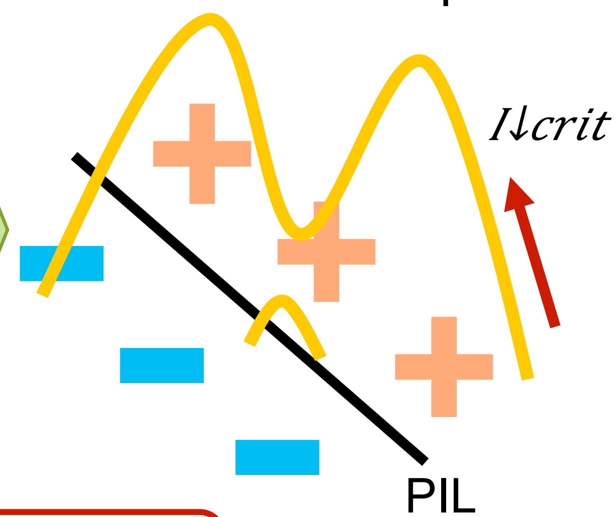
Twist criteria

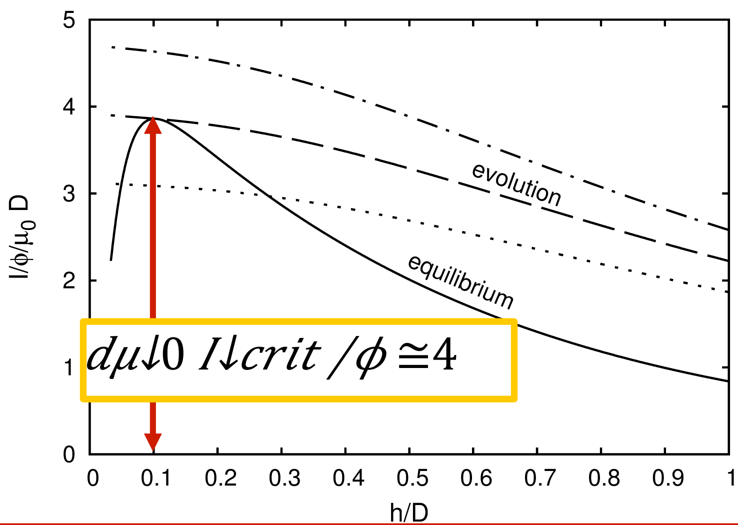
strongly sheared loop



injecting flux $\Phi_{rec} \propto I$ (L.F.F.F.)
&
reconnection

two-arc loop





$d\mu_0 I_{crit}/\phi \cong 4$

- L.F.F.F.

$\nabla\times B = \alpha B$

- Ampere's law

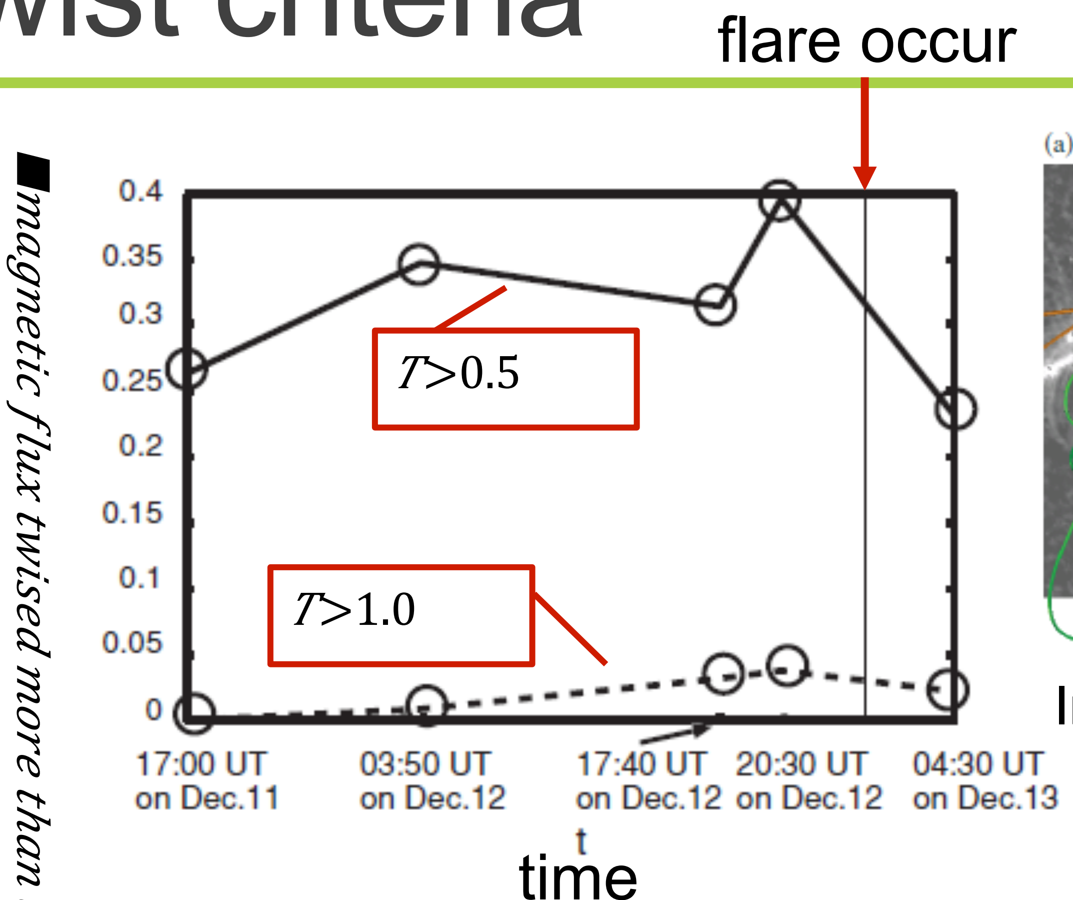
$\nabla\times B = \mu_0 J$

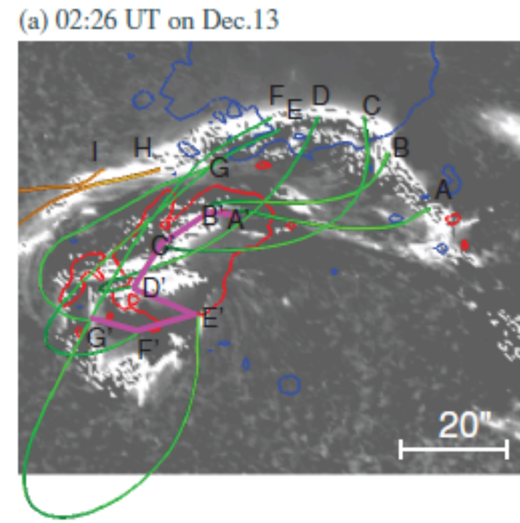
- The definition of twist

$T = \int \alpha \, dl/4\pi$

$T \gtrsim 1/2 \, \Phi_{rec}/\phi$
$\rightarrow T \gtrsim 0.5$

Twist criteria





Inoue et al. [2011]

Our result ($T \gtrsim 0.5$) is consistent with magnetic measurement by Inoue et al.

Summary

- We developed a numerical model to calculate the stability of two-arc flux rope.
- The critical height of instability of two-arc flux rope is much lower than that of torus instability.
- Under the two-arc constraint, decay index is not necessarily adequate criterion for the onset of eruption.
- More than half turn twist ($T>0.5$) of magnetic field is the necessary condition of the two-arc loop instability.

subslide

イントロ作成メモ

しかし不安定なトーラスに至るまでの過程はよく分かっていない、

例えばtether-cuttingモデルは強くシアした磁力線間でのリコネクションによりフラックスロープが不安定化されることを提案

これにconsistentなものが観測されており（最近ではChen et al. 2014）、またKusano et al. 2012の数値計算も同様にconsistentな結果を示している

でこのリコネクションによりtwo-arc形のループが形成されるわけだが、あんま安定性は議論されていない、形状が複雑なので

→これをやろう

question_memo

質問をかわす　That’s a good question. Let me think about that…

I need some time to consider that question. Could I discuss it with you after this session?

How say…

質問を別の表現で確認する→時間稼ぎでもある

質問の意味を尋ねる　I’m sorry, but I didn’t fully understand your question. Would you mind repeating it?

Could you repeat that question, please?

もっと大きな声で話すようたずねる　I’m sorry, I didn’t catch your question. Please could you speak a little louder?

とりあえずなんか言う　I‘m not sure but… ~~~

Take home message

question

■つなぎ目の構造はおかしくないのか？

■不安定解析の手法の説明が分かりづらい

■電流の強さが左右対称ではなかったら？Criteriaは？成長の仕方は？安定性は？

■今後の展望は？

■$\Phi\downarrow total=$の3本の線はどういうことか？

■uniform fieldの規格化はどうなってるのか？

■下半分はどうなってんの？

→電流は太陽表面の仮定から対称、ここからインダクタンスが決まる、一方で磁場は上しか考えていない、んでこの手法の正当性は軸対称なケースを仮定したDemoulinによる解析的な式を解いた先行研究の結果に一致することを確認している

question

■two-arcは形成直後からリコネクションにより外に向かう流れができているのでは？

■インダクタンスはどうやって求めているのか？

■点a, bの安定は電流が増えた（磁束量が変化した）場合には異なるのでは？

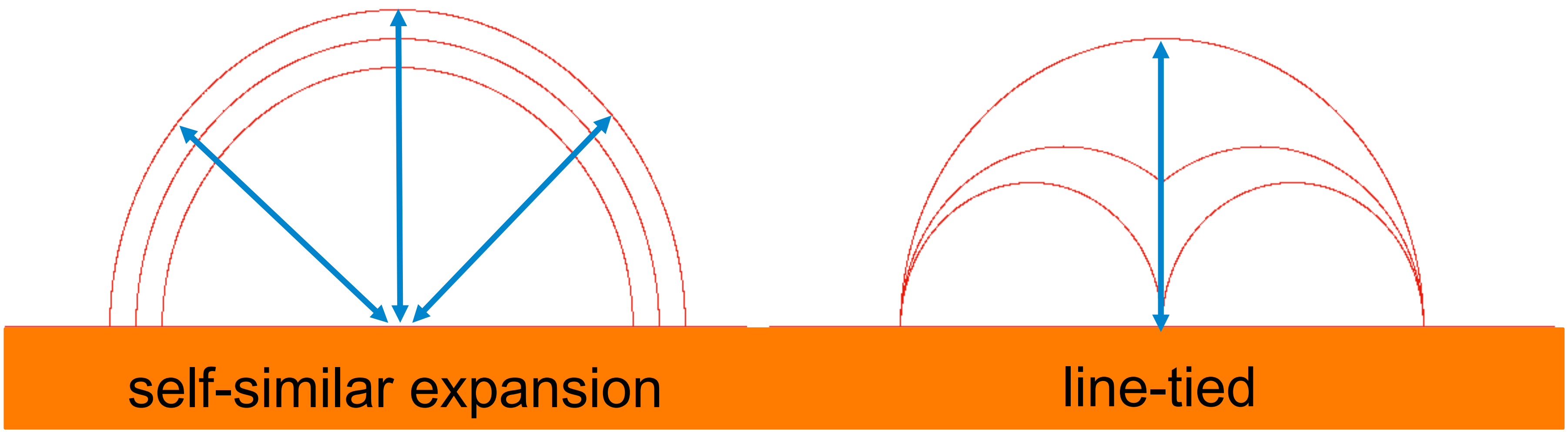
→一概には言えないが大きく注入されない限りまた別の安定平衡点に移行すると考えられる。

■電流のみを考えているが？フラックスロープという点では磁場を考えるべきではないのか？

→先行研究と同じくcurrent loop modelはforce-freeを考えているので磁場は考えなくてもよい

Torus Instability

Two-arc model



Because the time-scale for eruption phenomenon is shorter than the time-scale varying the magnetic distribution on photosphere, we should use line-tied assumption.

"d"の表現

ループの足の長さdの英語表現は

S_f is the distance between the fixed footpoints at the base of solar corona.

というのがChen, J. & Krall, J. [2003]のfigure 5の脚注に見られている。

Two-arcモデルと比較すると、$S_f=2d$という関係

Introduction (stability torus)

Demoulin & Aulanier [2010] analyzed the stability of axisymmetric current loop using these equations.

$$I_{eq}(h)=\frac{16\pi}{\mu_0}\frac{\phi D R (R^2+D^2)^{-3/2}}{\ln(8R/a)-1+l_f/2},\quad I_{evol}(h)=\left(\Phi-\iint B_{ex,x}\,dydz\right)\frac{2}{L}$$

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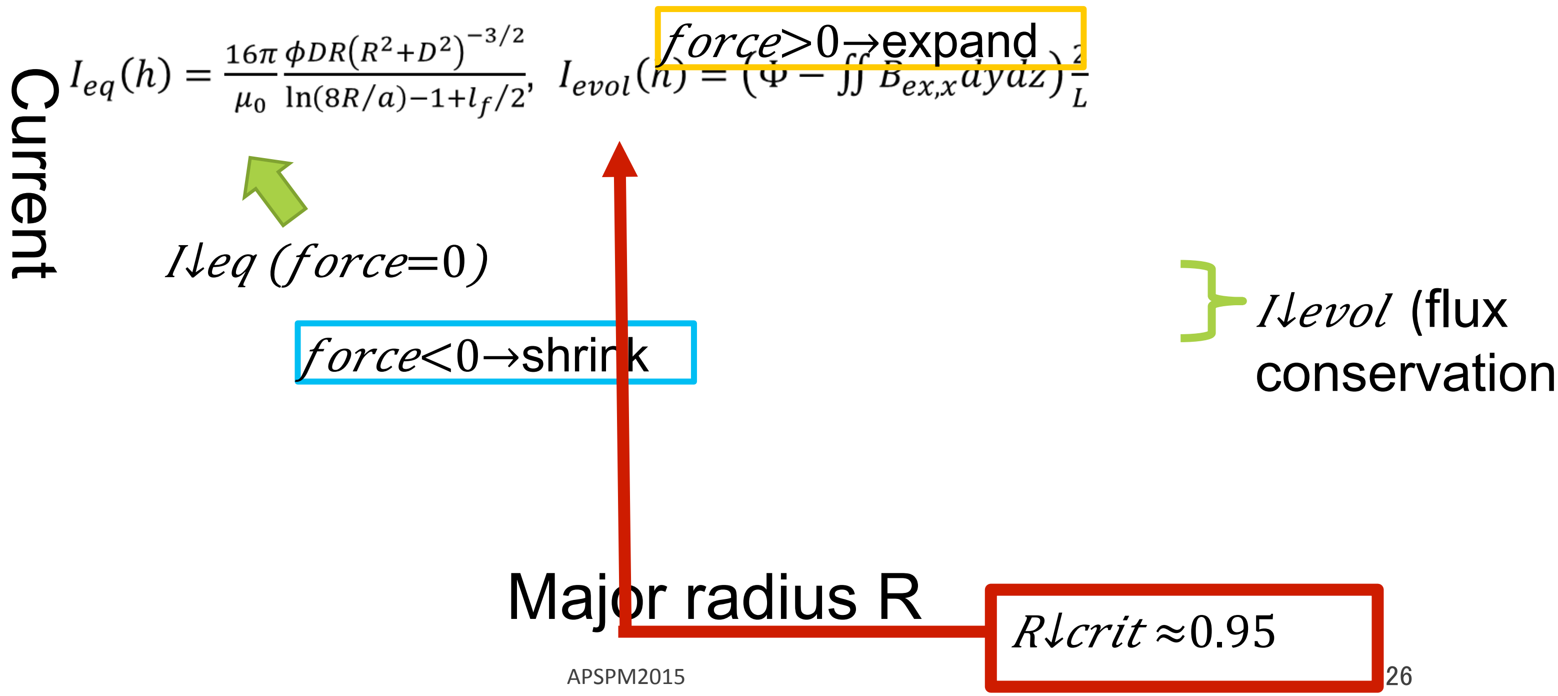
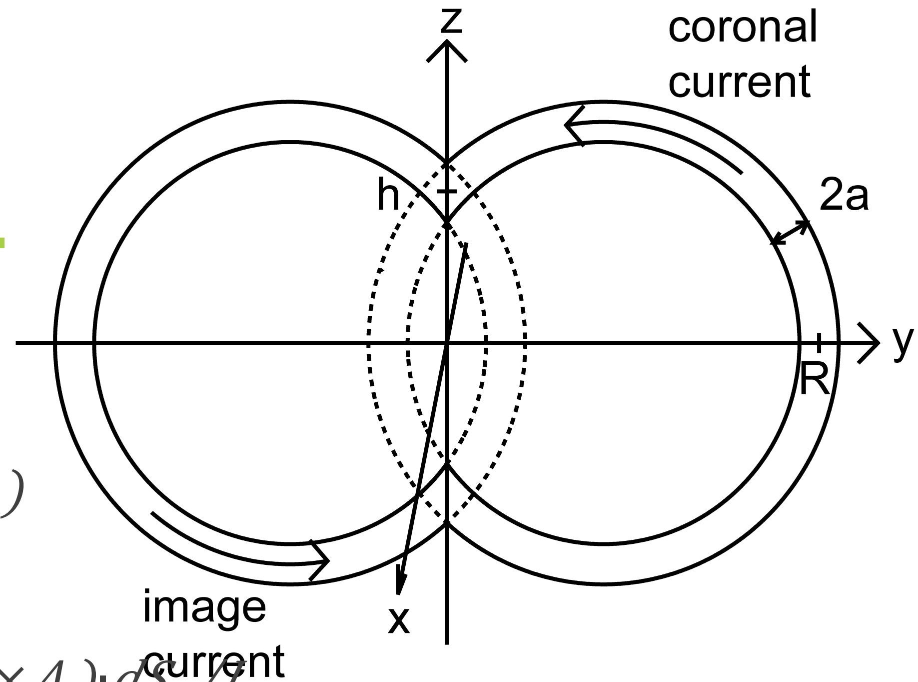
$$I_{eq}(h)=\frac{16\pi}{\mu_0}\frac{\phi D R (R^2+D^2)^{-3/2}}{\ln(8R/a)-1+l_f/2},\quad I_{evol}(h)=\left(\Phi-\iint B_{ex,x}\,dydz\right)\frac{2}{L}$$



Equations

$$I_{eq}(h) = -2 \frac{\partial \Phi_{ex}/\partial h}{\partial L/\partial h}$$

$$I_{evol}(h) = \frac{1}{L}\left(\Phi_{total} - \Phi_{ex}\right)$$



$$L_{ex} = \Phi/I = \int \mathbf{B}\cdot d\mathbf{S}\,/I = \int (\nabla\times\mathbf{A})\cdot d\mathbf{S}\,/I$$

$$= \oint \mathbf{A}\cdot d\mathbf{r}\,/I = \frac{\mu}{4\pi}\oint\oint \frac{1}{|\mathbf{r}-\mathbf{r}'|}\, d\mathbf{r}'\, d\mathbf{r}$$

$$L_{in} = \mu l/8\pi$$

$$L = L_{ex} + L_{in}$$

$$\Phi_{ex} = \oint \mathbf{A}_{ex}\cdot d\mathbf{r}$$

Decay index

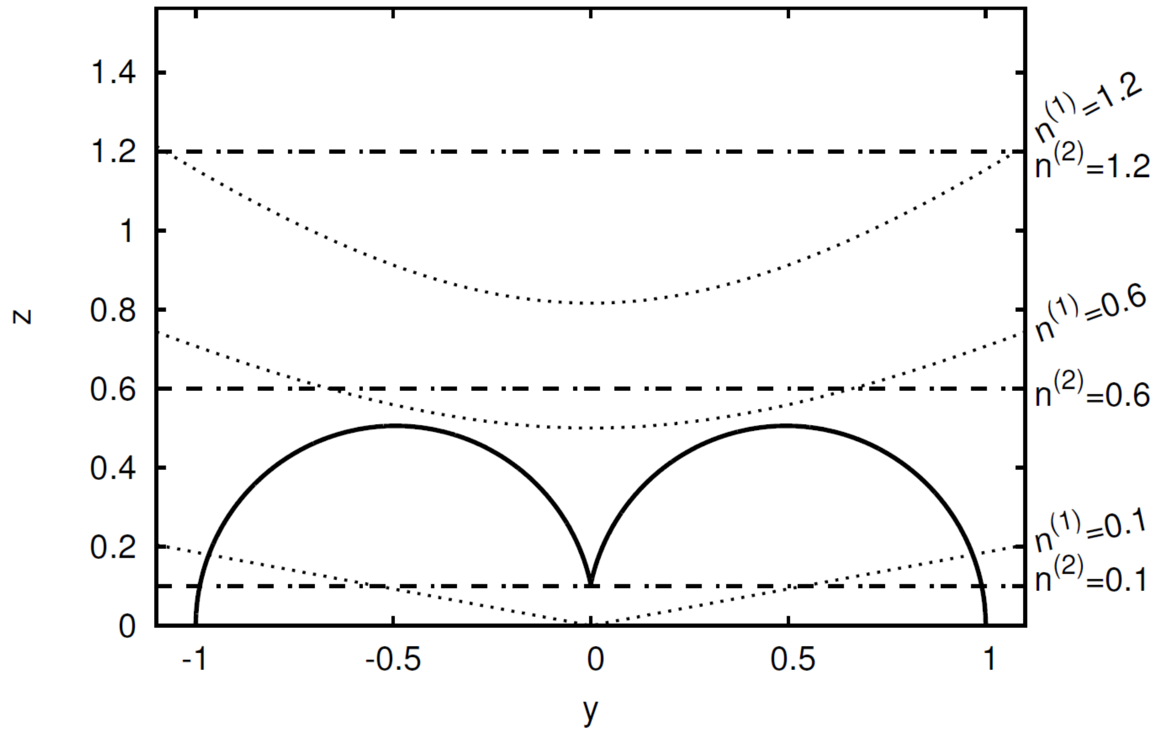
We check “decay index” under the two-arc loop.

Definition of decay index: $n \equiv -\partial \ln B_{ex} / \partial \ln |z|$

$$\rightarrow n = \begin{cases} 3z^2/(y^2+z^2+D^2) & \text{dipole} \\ |z|/z_0 & \text{exp} \\ 0 & \text{uniform} \end{cases}$$

Typically, critical decay index becomes $n > n_{crit} \approx 1.5$ in torus instability (Demoulin & Aulanier [2010]).

Decay index



○ The decay index on the loop is below about 0.6.

○ $n=0$ everywhere in the case of uniform field.

→ decay index is not necessarily an adequate criterion for the onset of eruption.

Twist criteria

We can derive the twist criterion for the onset of eruption from the critical current $I_{crit} \mu_0 D/\phi \cong 4$.

We assume that linear force-free field, use Ampere's law and definition of twist T for flux rope, then obtain following equation,

$$\Delta\Phi_{crit}/\phi \cong 1/2T$$

where $\Delta\Phi$ is flux injected to flux rope and ϕ is flux of external field.

From this relationship,

Derivation of twist criteria

critical current

$$I_{crit}\, \mu_0\, D/\phi \cong 4 \cdots (1)$$

Assuming linear force-free field,

$$\nabla \times B = \alpha B \cdots (2)$$

By Ampere's law,

$$I = \alpha \Phi / \mu_0 \quad \cdots (3)$$

From eq. (1), (3),

$$D \alpha \Phi / \phi \cong 4 \cdots (4)$$

The definition of twist

$$T = \int \alpha\, dl / 4\pi \cdots (5)$$

Loop length

$$l \cong \pi d/2 \cdots (6)$$

Substituting (6) for (5),

$$\alpha \cong 8T/d \cdots (7)$$

Here, we define $D = d$,

$$\Phi_{rec}/\phi \cong 1/2T$$

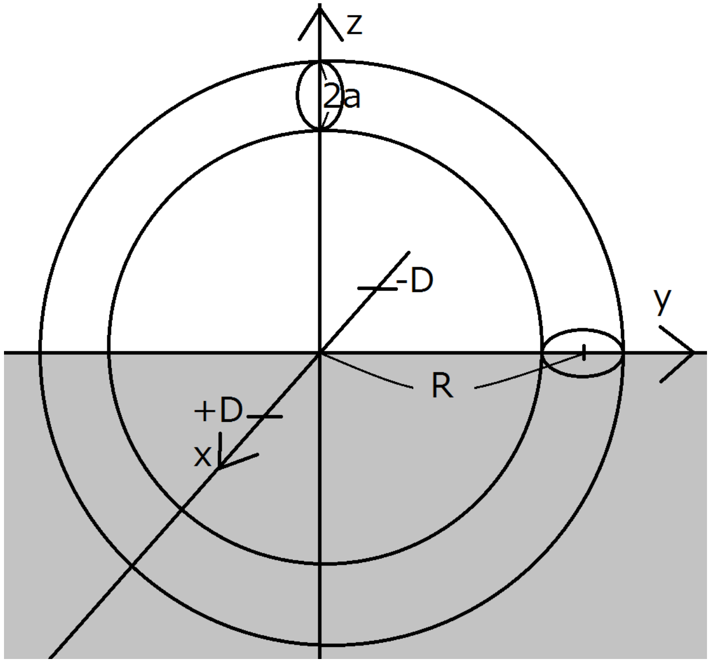
Simulation-1 (Torus)

First, we calculate for **axisymmetric** flux rope,

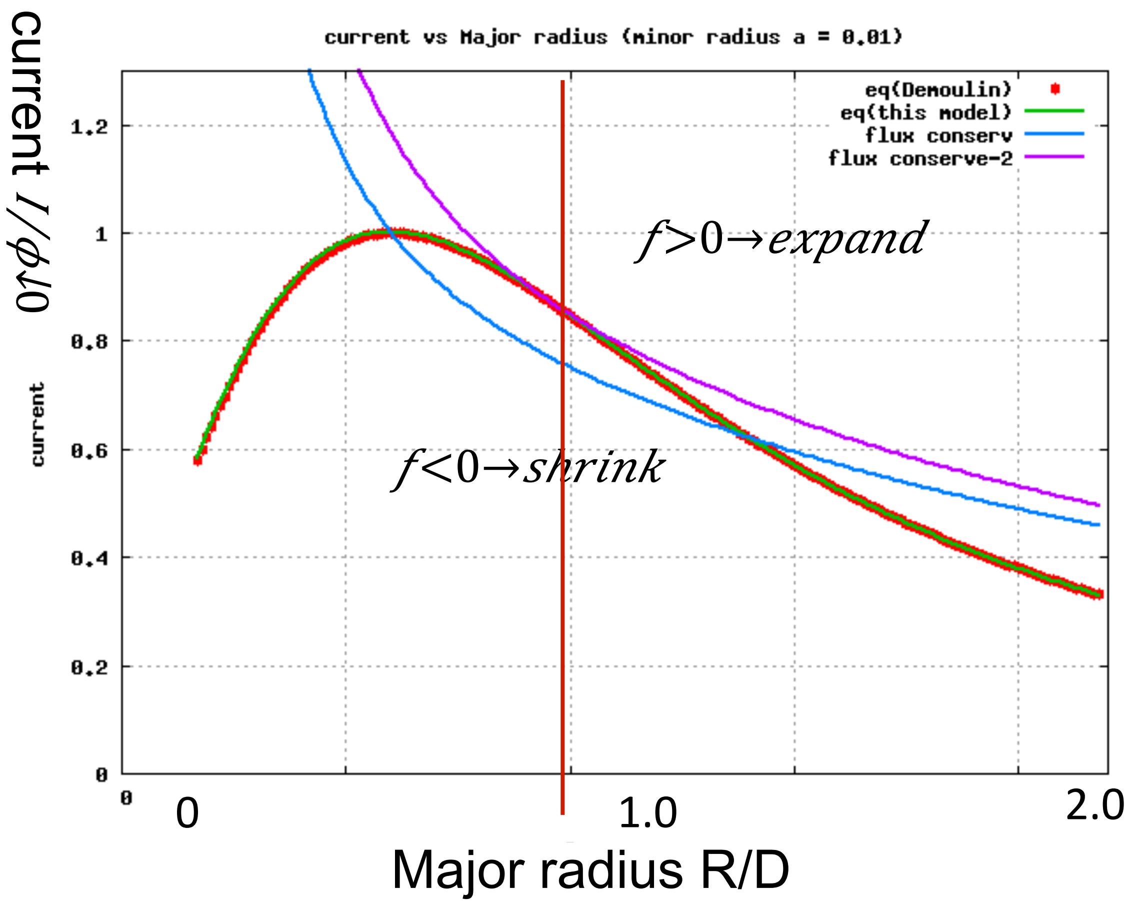
- to compare with the Demoulin result
- to check whether this model is able to calculate correctly

Length is normalized by the D.

Parameter		
	Setting 1	Setting 2
Toroidal grid	1000	1000
Poloidal grid	10*30	10*30
minor radius (a)	0.01	0.1
Major radius (R)	0.15~2.2	0.15~2.2
Aspect ratio(=R/a)	15~220	1.5~22



Result-1 (code check)



$\rightarrow R_{loss\ of\ equilibrium} \cong 1.0$

Red point: equilibrium curve for Demoulin equation

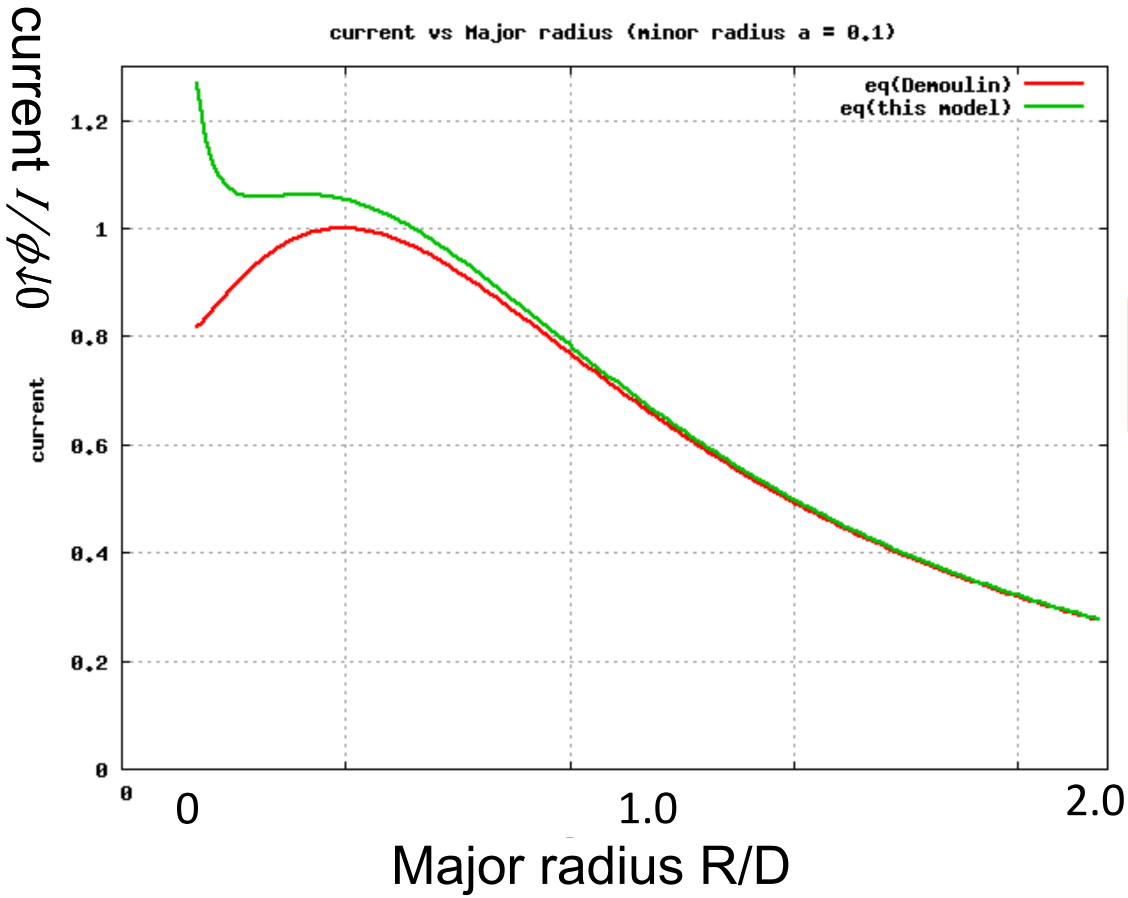
Green line: equilibrium curve for this model

Blue line: evolution curve for $\phi_0 = 1.0$

Purple line: evolution curve for $\phi_0 = 0.766$

Toroidal grid	1000
Poloidal grid	10*30
minor radius (a)	0.01
Major radius (R)	0.15~2.2
Aspect ratio(=R/a)	15~220

Result-2 (code check)



Red line: equilibrium curve for Demoulin equation

Green line: equilibrium curve for this model

→When the aspect ratio $R/a \gtrsim 10$, this model is validated.

Toroidal grid	1000
Poloidal grid	10*30
minor radius (a)	0.01
Major radius (R)	0.15~2.2
Aspect ratio(=R/a)	15~220